

Chris: Barnatt
His Book

1776. 1776.

Thomas Sturge &

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Stereometry,
Or, The ART of
G-A U G I N G

Made easie, by the HELP of a
SLIDING-RULE:

Which shews the Area's of Circles in Gallons, and the Square and Cube-Root of any Number under 100000, by Inspection: And is also very useful in *Arithmetick* and *Geometry*, particularly in the Measuring of *Superficies* and *Solids*.

WITH AN
A P P E N D I X
Of **CONICK SECTIONS:**

Shewing the *Nature*, *Properties*, and *Contents* of several *Solids*; frequently mentioned in Books of **GAUGING**.

The Eighth Edition, carefully Corrected.

To which are added several New **TABLES**, with Short and Plain Rules for Gauging of Malt.

By **THO. EVERARD, Esq.**

L O N D O N:

Printed for *Mary Hussy* in *Little-Britain*, *B. Tooke* at the *Temple-Gate*; *R. Wilkin*, *D. Midwinter*, *B. Cowse*, and *G. Mortlock*, in *St. Paul's Church-Yard*. 1721.

SALE

OF THE

LANDS

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TO THE

Right Honourable

SIDNEY

LORD *Godolphin*,

L A T E

Lord HIGH-TREASURER

O F

ENGLAND;

This TRACT is Humbly

Dedicated and Presented

B Y T H E

A U T H O R.

TO THE

Right Honourable

SIR

Lord Godolphin

AT

Lord High Treasurer

OF

ENGLAND

The TRACT is Humbly

Dedicated and Presented

BY

AUTHOR

THE
PREFACE

THE Instrument described in this Tract, (tho' useful to all Persons concern'd in the Measuring of Superficies and Solids) was principally design'd for the Assistance of the Officers of the Excise; for whom it is now found so useful and necessary, that no

The P R E F A C E.

Gauger is thought to be well prepared for his Business without it : But the many Thousands that have been Sold, and the several Editions of this Tract, are so good Evidence of its Utility, that it is as needless, as unbecoming for me to say any thing in its Commendation.

If the READER finds this Edition somewhat different from some of the former, it is presumed he will have no cause to complain of the Alterations ; for besides, that this is more Portable, and more Correct, here is also added (Sect. IX.) some Short and Plain

The PREFACE.

Plain Rules for the Gauging of
Malt; with a particular Account
of the Dimensions and Content
of the Standard-Bushel, and how
the same were found and ascer-
tained; also a Table shewing the
Diameter and Depth of any
legal Bushel, Half-Bushel,
Peck, or Gallon, with Directi-
ons for making any of these Mea-
sures, and for Rectifying such
as are already made; with seve-
ral other useful Tables, not hi-
therto made Publick; all which
have render'd this Tract so
compleat, that it is presumed
there will be no occasion to Alter
or Add to it in any future Edi-

The PREFACE.

tion: *I have only this further to Advertise the Reader of, viz.*

That the Principal Lines upon this Instrument are put upon a Tobacco-Por, to slide one by another, as they do upon the Rule, but with this considerable Advantage, that here the whole of one Line is always applied to the whole of another; so that all the Questions which may be resolved by these Lines on the Rule, may be resolved more readily by the said Lines on this Por, and the Directions given in this Tract for the Use of the Rule, are also applicable to the

The PREFACE.

the respective Lines upon the Box.
I shall Instance in one particular,
which may serve instead of many;
and that shall be in finding the
Area of a Circle in Ale-Gal-
lons, suppose the Diameter be 40
Inches.

Set the Gage-Point for Ale-
Gallons upon D, to 1 upon the
Line C; then against 40 upon
D, you have 4.45 upon C, which
is the Area required; and the
Lines being thus set, you have
in effect, a Table of the Area's
of Circles in Ale-Gallons, to
all Diameters whatsoever, with-
out moving the Lines backward or
for-

The PREFACE.

forward, as must be done on the Rule when one Line is drawn out beyond the End of the other.



Adver-



ADVERTISEMENT.

THE *Sliding RULE* mention'd in this Book, and all Sorts of *Gauging Instruments*, useful and proper for His Majesty's Officers of the Customs; are very exactly and accurately made by *Jacob Carver*, Son to *Isaac Carver*, the original Maker, at the *Atlas and Quadrant* near *Cherry-Garden Stairs*, on *Rotherhith-wall*, London.

†† All Sorts of *Gauging* and *Mathematical Instruments*, in *Silver*, *Brass*, *Ivory*, and *Wood*, made and Sold by *Tho. Cook*, *Mathematical Instrument-maker*, by the *Excise-Office* in the *Old-Jury*.





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THE INTRODUCTION OF

Decimal Arithmetick.

WHOWER will rightly understand the Art of Gauging, or the Use of the Instrument here described, ought in some measure to be acquainted with the Art of *Arithmetick*; at least with *Numeration*, *Addition*, *Substraction*, *Multiplication*, and *Division*, both in *whole Numbers* and *Decimal Fractions*.

And forasmuch as most of the *Problems* in this *Tra&* are resolved by help of the *Line of Numbers*, the several *Uses* of which *Line* are render'd more easie by the knowledge of *Decimal Arithmetick*: I shall in the first place endeavour to shew what a *Decimal Fraction* is, and also give some *Rules* and *Examples* by which any Person who does at all understand the vulgar *Arithmetick*, may in a few hours time thoroughly comprehend this.

A *Decimal Fraction* is that, which by prefixing a Point or a Prick towards the left hand, its Value is decreas'd from so many *Units*, to so many tenth Parts of an *Unit*; and if a Point and a Cypher, or Digit be prefixed, it will then be so many hundred

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parts,

parts, and if a Point and two Cyphers or Digits be prefixed, its value is decreased to be but so many thousand parts: As, if you would prefix before the Figure 2, a point thus $[\cdot 2]$ 'tis then decreas'd from 2 Units, to 2 tenth parts of an Unit. For,

As in *whole Numbers* the Value or Denomination of places *increases* by Tens from the Unit's place towards the *left hand*; so in *Decimal Fractions*, the Value or Denomination of Places *decreases* by Tens from the Unit's place towards the *right hand*; as in the following Scheme.

In which I have set an Unit in its due place, and all the Figures towards the left hand (being whole Numbers) do *increase* by Tens; for the Figure 2, next the Unit's place is 2 Tens (or 20,) the next is 3 hundreds, and the next 4 thousands, &c.

Hundreds of Thousands.	Ten of Thousands.	Thousands.	Hundreds.	Tens.	Unit.	Tenth parts.	Hundred Parts.	Thousand Parts.	Ten Thousand Parts.	Hundred Thouf. Parts.
6	5	4	3	2	1	.2	3	4	5	6

But all the Figures from the Unit's place, towards the right hand (being *Decimal Fractions*) do *decrease* by Tens; so the Figure 2, towards the right hand is 2 tenth parts, the next is 3 hundred parts, and the next 4 thousand parts of an Unit; each place towards the right hand being ten times less.

Decimal Arithmetick.

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In *Decimal Arithmetick*, we always imagine (and it would be very commodious if it were really so) that all intire Units, Integers, and Things, are divided into ten equal parts, and each of these parts so divided we call *Primes*; we also divide each of these *Primes* into other ten equal parts, and every of these Divisions we call *Seconds*; and each of these being divided into ten other equal parts, may be called *Thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, so one Second is $\frac{1}{100}$ of $\frac{1}{10}$ or $\frac{1}{1000}$ of an Unit, and one third is $\frac{1}{1000}$ of $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{10000}$ part of an Unit) so by dividing the former, and subdividing these latter, we may run on *ad infinitum*.

Let a Pound Sterling be given to be Decimally divided :

According to the Notion premised, the first Division must be *Primes*, the next Division *Seconds*, the next *Thirds*, &c.

So if one Pound Sterling (being 20 s.) be divided into ten equal parts, one of these parts is one *Prime*, and its value is 2 s. and will stand thus [.1] three *Primes*, or three *tenths* of a Pound will stand thus [.3] and its value is 6 s. Again, one *Prime*, or 1 being divided into ten equal parts, each of those parts will be one *Second*, or one hundredth part of a Pound, and is thus express'd [.01] and its Value will be 2 d. 1 f. and $\frac{6}{100}$ of a Farthing; and if .01 be divided into ten other equal parts, each of those parts will be *Thirds*, or one *thousandth* part of a Pound, and will stand thus, [.001] and its Value will be $\frac{26}{1000}$ of a Farthing.

The like may be understood of one Pound *Troy* or *Averdupois*, one Foot, one Gallon, or any other Integer, or Thing whatsoever.

A *Decimal Fraction*, whether it stand alone or be joined with *whole Numbers*, is distinguished from a *whole Number* by a Point or Prick before it, towards the left hand, as in these Examples.

.4752 4.752 47.52 475.2

In *Decimal Fractions* the *Numerators* only are set down, the *Denominators* being known by the *Number* of Places in the *Numerator*; for if the *Numerator* consist of but one Place, the *Denominator* is 10, if of 2 places, the *Denominator* is 100; &c.

Example.

The *Denominator* of $\left. \begin{array}{l} .2 \\ .25 \\ .051 \\ .0752 \end{array} \right\}$ is $\left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \right\}$

As Cyphers *before* a *whole Number* have no Value, so Cyphers *after* a *Decimal Fraction* are of no signification, and therefore will not increase the Fraction, nor alter the *Denominator*; for .2 is two tenth parts, and .20 is no more.

Again, as Cyphers *after* a *whole Number* do increase that Number, so Cyphers *before* a *Decimal Fraction* do diminish the Value thereof; as in these Examples.

$\left. \begin{array}{l} .5 \\ .05 \\ .005 \\ .0005 \end{array} \right\}$ is 5 $\left\{ \begin{array}{l} \text{Tenth} \\ \text{Hundred} \\ \text{Thousand} \\ \text{Ten Thousand} \end{array} \right\}$ Parts.

Here

Decimal Arithmetick.

Here every Cypher added removes the Fraction further from Unity, making it ten times less than before.

Addition and Subtraction of Decimal Fractions.

AS for the Operation of Addition and Subtraction in Decimals, it is the very same with the Vulgar; there must only care be had of placing Units under Units, and Fractions under Fractions in their proper Ranks and Files.

For Example.

To 376.435
Add 54.22

Sum is 430.655

Again,

To 26.58
Add 527.342

Sum is 553.922

Subtraction.

$$\begin{array}{r}
 \text{From} \quad \text{—————} \quad 430.645 \\
 \text{Subtract} \quad \text{—————} \quad 54.210 \\
 \hline
 \text{Remains} \quad \text{—————} \quad 376.435
 \end{array}$$

Or, thus:

$$\begin{array}{r}
 \text{From} \quad \text{—————} \quad 553.852 \\
 \text{Subtract} \quad \text{—————} \quad 527.342 \\
 \hline
 \text{Remains} \quad \text{—————} \quad 26.510
 \end{array}$$

Multiplication of Decimals.

1. **I**N *Multiplication* both of *Decimal Fractions* and *mixt Numbers*, there are ever *two* Numbers given to find a *third* unknown.

One of the Numbers given (and it's no matter which) is called the *Multiplicator*, the other the *Multiplicand*, and the Number sought is called the *Product*; and this doth ever contain one of the Numbers given, as many times as the other given Number contains Unity.

Multiplication of Decimals is performed after the *same manner* as in *whole Numbers*; for the Numbers being set down one under another, we proceed in the *Multiplication* as if they were all *whole Numbers*; only it must be observed (when the *Operation* is finished) how *many* places of *Decimal Fractions* are contain'd in the *Multiplicator* and *Multiplicand*, for so *many* must be in the *Product*, as in these Examples.

Mul-

Decimal Arithmetick.

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$$\begin{array}{r} \text{Multiplicand} \text{ --- } 35.25 \\ \text{Multiplier} \text{ --- } 7.24 \end{array}$$

$$\begin{array}{r} 14100 \\ 7050 \\ 24675 \end{array}$$

$$\text{Product} \text{ --- } 255.2100$$

Or, thus :

$$\begin{array}{r} \text{Multiplicand} \text{ --- } 5.75 \\ \text{Multiplier} \text{ --- } .25 \end{array}$$

$$\begin{array}{r} 3375 \\ 1350 \end{array}$$

$$\text{Product} \text{ --- } 1.6875$$

2. When it happens (as it often does) that there are not so many *Figures* in the *Product*, as there are *Fractions* in the two *Numbers* to be multiplied, you are then to place *Cyphers* before the *Product* till the *Number of Places* be equal ; as in these Examples.

$\begin{array}{r} .04 \\ .6 \\ \hline .024 \end{array}$	$\begin{array}{r} .75 \\ .05 \\ \hline .0375 \end{array}$	$\begin{array}{r} .85 \\ .005 \\ \hline .0425 \end{array}$
---	---	--

3. As *whole Numbers* are increased by *Multiplication*, so *Decimal Fractions* are hereby made less ; for the *Product* is removed farther from *Unity* than either of the *Fractions* given to be multiplied ; as appears by the last Examples.

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4: When

4. When a *Decimal Fraction* or *mixt Number* is to be multiplied by an Unit with Cyphers (as 10, 100, 1000, &c.) you need only remove the Point so many places towards the right Hand, as there are Cyphers with the Unit; thus, if .7562 be multiply'd

$$\text{By } \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \right\} \text{ The Product will be } \left\{ \begin{array}{l} 7.562 \\ 75.62 \\ 756.2 \\ .7562 \end{array} \right\}$$

Division of Decimals.

1. IN *Division* there are *two Numbers* given, to find a *third unknown*.

One of the Numbers given is called the *Divisor*, the other the *Dividend*, and the Number sought is called the *Quotient*, and this *Quotient* doth ever contain Unity as many times as the *Divisor* is contained in the *Dividend*.

Division of Decimal Fractions or *mixt Numbers*, is performed after the same manner as in whole Numbers; all the Difficulty is to know the Denomination of places in the *Quotient*, and they are easily discovered by this one general Rule.

The first Figure in the *Quotient* must be of the same Denomination with that Figure in the *Dividend*, which at the first demand stands exactly over the place of Unity in the

Divi-

Divisor. I shall make this plain by some Examples.

Example.

Let it be required to divide 32 by 51.2.

In this Example the *Divisor* (51.2) is greater than (32) the *Dividend*, therefore in this and the like cases you must place a competent Number of Cyphers behind the *Dividend*, which (if it be a whole Number) the Cyphers so placed must be distinguished there-from as Fractions; as here I put 4 Cyphers behind 32, and placing the Numbers in this manner,

$$51.2 \overline{) 32.0000}$$

I proceed to the Division (as if all were whole Numbers) and first ask, how many times the *Divisor* is contained in (320) the three first Figures of the *Dividend*? And seeing in this Ex-

$$51.2 \overline{) 32.0000} \begin{array}{r} 6 \\ 128 \end{array}$$

ample it is not contained in the three first Figures, it will therefore extend to the fourth place, over which I put a prick, to shew that I must begin to subtract at that place. I then ask how many times 5 in 32? I find 6, I place 6 in the *Quotient*, and multiply the *Divisor* by 6, Subtracting the Product out of the *Dividend*, beginning my Subtraction at the place over which I put the prick: Thus 6 times 2 is 12, 12 from 0 I cannot, borrow 2, and say 12 from 20, and there remains 8, which I place under the Cypher: Again, 6 times 1 is 6, and 2 I borrowed is 8, 8 from 10, and there

B 5

there rests 2, which I place under the next Cypher before 8, as in the Example; then 6 times 5 is 30, and 1 I borrowed is 31; 31 from 32, and there rests 1, which I set under 2; then for my new Dividend I bring down the next Cypher, and set it behind the Remainder, and the Example will stand thus:

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (6 \\ 1280 \end{array}$$

Then proceeding in my Division, I ask how many times 5 in 12, I find 2, which I place in the *Quotient*, and say 2 times 2 is 4; 4 from 10, and there remains 6, which I set under 0: Again, 2 times 1 is 2, and 1 I borrowed is 3; 3 from 8, and there rests 5, which I place under 8, then 2 times 5 is 10; 10 from 12, and there rests 2, which I place under 2, and to this Remainder I bring down another Cypher to make my new *Dividend*; so will the Example stand thus:

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (62 \\ 1280 \\ 2560 \end{array}$$

This done, I enquire how many times 5 in 25, and find 5, which I place in the *Quotient*, and say, 5 times 2 is 10, 10 from 10 and there rests 0; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6, and there rests 0; Lastly, 5 times 5 is 25, 25 from 25 and there remains 0; and thus my

Decimal Arithmetick.

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my Division is finished, and the Example will stand thus :

$$\begin{array}{r} 51.2 \overline{) 32.0000} \quad (.625 \\ 1280 \\ \underline{.2560} \\ 0000 \end{array}$$

Now by the general Rule above given, it is plain that the first Figure in the *Quotient* must be Tenths, and therefore the *Quotient* is all Decimal parts ; for the *Divisor* being placed under the first Figures of the *Dividend*, this Example will stand thus :

$$\begin{array}{r} 32.0000 \quad (.625 \\ 51.2 \end{array}$$

By which it appears, that the *Cypher* in the *Dividend*, which stands over the *Unit's place* in the *Divisor*, is the place of *Tenths*.

See more Examples.

Divisor. *Dividend.* *Quotient.*

$$51.2 \overline{) 320.000} \quad (.625$$

$$52.2 \overline{) 32.0000} \quad (.625$$

$$7.5 \overline{) 375} \quad (.95$$

2. In Division of whole Numbers or *mixt*, if there happen to be a Remainder, you may bring down more *Cyphers*, and by continuing your Division, carry on the *Quotient* to as many places of *Fractions* as you please ; four or five will be very near the truth, but two or three will be sufficient in most Cases.

Example ;

Example.

6.26) 25.800000 (4.1214

760

1340

880

2540

36

3. When any Number (either *Decimal* or *mixt*) is to be divided by an Unit with Cyphers, as 10, 100, 1000, &c. you need only remove the prick in the *Dividend* so many places toward the left Hand, as there are Cyphers with the Unit, supplying the vacant places (if any be) with Cyphers.

Thus 756.2 divided by 10, is 75.62, divided by 100, it will be 7.562, by 1000, .7562, and if by 10000, the *Quotient* will be .07562.

4. *Division* is proved by *Multiplication*; and *Multiplication* by *Division*.

To prove *Division*, multiply the *Quotient* by the *Divisor*, and the *Product* is the *Dividend*: Thus, .652 (the *Quotient* in the first Example of *Division*) being multiplied by 51.2 the (*Divisor*,) the *Product* is 32.0000. (the *Dividend*.)

To prove *Multiplication*, divide the *Product* by either of the Numbers given to be multiplied, and the *Quotient* will be the other Number given.

Thus if the last *Product* (viz.) 32.0000 be divided by 51.2, the *Quotient* is .625, or of 32.0000 be divided by .625 the *Quotient* will be 51.2.

And hence 'tis evident, that *Division* is also proved by *Division*. For if any *Quotient* be made a *Divisor*, it will quote the first *Divisor*. Example:

1.8)

$$1.8) 28.8 (16$$

$$\begin{array}{r} 168 \\ 000 \end{array}$$

$$16) 28.8 (1.8$$

$$128$$

$$000$$

5. Now forasmuch as *Multiplication* is easier than *Division*, I shall here shew, that what is performed by a *Divisor*, may also be performed by a *Multiplier*, and how such *Multiplier* may be found: Suppose I were to divide 900 by 25, the *Quotient* will be 36.

Now let it be required to multiply 900 by a certain Number that shall produce 36.

To find this *Multiplier*, divide an Unit with Cyphers by 25, the *Divisor* proposed; the *Quotient* will be .04 the *Multiplier* sought; for 900 being multiply'd by .04, the *Product* will be 36, which was required.

Again, Having a *Multiplier*, to find a *Divisor*, this is but the *Converse* of the former; for if an Unit with Cyphers be divided by .04 (a *Multiplier*) the *Quotient* will be 25, the *Divisor* as before.

6. A *Vulgar Fraction* is reduced into a *Decimal* by *Division*: For which the Rule is,

Divide the *Numerator* (of the *Vulgar Fraction* given) by the *Denominator*, and the *Quotient* will be a *Decimal Fraction*, equal in Value to the *Vulgar Fraction* given: So $\frac{1}{4}$ reduced as aforesaid will be .25: *Example*.

$$4) 3.00 (.75$$

$$\begin{array}{r} 2000 \\ 00 \end{array}$$

Note

Note, The *odd Parts* of an Integer cannot be exactly reduc'd to a *Decimal*, for there will always be a *Remainder*; in such Cases carry on the Fraction to four or five places, it will be very near the Truth. *Example*;

$$\begin{array}{r} 4 \\ 9 \overline{) 4.0000} \end{array} \begin{array}{l} 9) 4.0000 \text{ (.4444} \\ 40 \\ 40 \\ 40 \end{array}$$

7. To reduce a *Decimal Fraction* to the *known parts* of the *Integer* :

If the *Decimal* given be part of a *Pound Sterling*, multiply it by 20 (the *Shillings* in a *Pound*) and the Fractions in the Product by 12 (the *Pence* in a *Shilling*) and the Fractions in this Product by 4 (the *Farthings* in a *Penny*.) Then the *whole Numbers* in the respective Products shew the *Shillings*, *Pence* and *Farthings* contained in the *Decimal* given, and the *Fractions* in the last Product are *Decimals* of a *Farthing*. *Example*; Suppose .25 a *Decimal Fraction* of a *Pound Sterling*, were given to be reduc'd as aforesaid: Multiply it by 20, the Number of *Shillings* in a *Pound*, the Product will be 5, viz. 5 s.

$$\begin{array}{r} .25 \\ 20 \overline{) 5.00} \\ \hline \text{Shillings} \end{array}$$

So the *Value* of .60625 will be found to be 12 s. 1 d. As appears by the following *Example* :

.60625

Decimal Arithmetick.

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	.60625
	20
Shillings	12.12500
	12
	25000
	12500
Pence	1.50000
	4
Farthings	2.00000

If the Decimal given be part of a *Beer Barrel*; multiply it by 4, (the *Firkins* in a Barrel) and the Fractions in the Product by 9 (the *Gallons* in a Firkin,) and the Fractions in that Product by 8, (the *Pints* in a Gallon.) The whole Numbers in the respective Products are equal to the *Firkins*, *Gallons* and *Pints* contained in the Decimal propounded.
Example.

Let .72 be a Decimal of a Beer Barrel to be reduced as aforesaid, it will be found to be 2 Firkins 7 Gallons, 7 Pints, and .36 of a Pint, as appears by

	.72
	4
Firkins	2.88
	9
Gallons	7.92
	8
Pints	7.36

But

But *these* and *many other* Arithmetical Questions may be more *readily resolved* by the *Instrument*; a Description of which see in the first Section of the following Discourse.

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It is the intention of the Author to give a full and complete Description of the Instrument, and to show the manner of using it in all the Operations of Arithmetick, and to give a full and complete Description of the Instrument, and to show the manner of using it in all the Operations of Arithmetick.

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THE

STEREOMETRY,

OR, THE

ART of GAUGING

MADE EASIE

By the HELP of a New

SLIDING-RULE.

SECT. I.

Description of the Rule, and of the several Lines upon it, with their Use in some Questions in Arithmetick.

THE Rule consists of Three Parts, viz. a Rule of 12 Inches long, and two small Scales to slide in it, which may be drawn out, one towards the right Hand, and the other towards the left, till the whole be three Foot long. The principal Lines on the Instrument are those commonly known by the Name of Gunter's-Line,

or

or *Line of Numbers*, which are here distinguished one from another by certain Letters, set at the end of the Lines towards the right hand.

Thus the Lines D, are each of them one *single Line of Numbers*, beginning at the End of the Rule towards the left hand, and from thence continued to the other End.

The Lines AB, and C, are called *double Numbers*, each being *two* Lines or Radius's of Numbers, the *first* beginning at the *left* hand, and ending in the *middle* of the Rule where the *second* Radius begins, and is from thence continued to the *end* at the *right* hand.

The Line E is call'd *Triple Numbers*, being *three* Radius's of Numbers; the *first* beginning at the *left* hand, and the *third* ending at the *right* hand.

This *Triple Line* is *equal* in length to the *double Lines*, and all to the *single Line*; for all the *five* begin and end at the *same Point*.

The *Line of Numbers* is (now) so well known to most Persons, that it may be thought a *sufficient Description* to have only said that *these* are *such*.

But for as much as this Instrument may be useful to some who (I presume) do not yet know what the *Line of Numbers* is, I shall therefore endeavour to explain it as followeth.

The *Line of Numbers*, is a Line of Geometrical Proportions, divided first into *Nine unequal Parts*, call'd *Primes*, which are distinguish'd by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of these *Primes* are subdivided into *ten* other parts (according to the same Reason) call'd *Tenths*, and again each of those *Tenths* are subdivided, or at least suppos'd to be subdivided into *ten* other parts, according as the length of the Line will admit; as here the *Line D*, being about 11 Inches long, each tenth in the first *Prime*

is really subdivided into *ten* parts, call'd *Centesms*, but betwixt the Figures 2 and 4, each *tenth* is divided but into five parts, and therefore each of those parts do signifie *two Centesms*: Again, from the Figure 4 to the end of the Line, every tenth is divided into two parts, each representing five *Centesms*: Lastly, each of these *Centesms* is also supposed to be divided into *ten* parts, which by some are called *Millions*; but a Line of this Length will not admit of this last division.

The Figures (1, 2, 3, 4, 5, 6, &c.) by which the *Primes* are distinguish'd, are all arbitrary Points, and may any of them represent so many entire *Units*, *Tens*, *Hundreds* or *Thousands*; or they may also represent so many *Tenth*, *Hundredth*, *Thousandth*, or *Ten Thousandth* parts of an *Unit*.

1. For *whole*, or *intire Units*; Let the first *Prime* or the Fig. 1. at the beginning of the Line D, represent one *Unit*, then shall all the Figures towards the right hand, (*viz.* 2, 3, 4, 5, &c. to 10.) represent so many *Units*, and the tenths in each *Prime*, will be *tenth parts*, and the *Centesms* in each of those tenths will be *hundredth parts* of an *Unit*.

Or, let 1. at the beginning of the Line represent 10 *Units*, then will each *Prime* forwards represent 10 times so many *Units* as the Figures express; thus, the Figures 2, 3, 4, 5, &c. will be 20, 30, 40, 50, &c.

And when one *Prime* represents 10 *Units*, every *tenth* in that *Prime* will be 1 *Unit*, and each *Centesm* in those tenths will be 1 tenth part of an *Unit*.

Again, Let the first *Prime* represent 100, then the Figures 2, 3, 4, 5, &c. will represent 200, 300, 400, 500, &c. and therefore 10 at the end will be 1000; and according to this Supposition, 1 tenth in each *Prime* will be 10 *Units*, and in those tenths each *Centesm* will be 1 *Unit*.

2. For

2. For *Decimal Fractions*; Let 10 at the end of the Line (at D) represent 1, then each *Prime* towards the left hand will be .1, and in those *Primes* each tenth will be .01, and in these tenths each *Centesm* will be .001 part of an Unit.

To make this more plain, draw out the sliding Piece B, till 1 at the *beginning* of the Line B, stand exactly against 10 at the *end* of the Line A, for then you have a Line of Numbers four times repeated upon which, let 1 at the beginning of the Line A represent 1 Unit; then shall 1 in the *middle* of the said Line be 10; and 10 at the *end* thereof (or which is all one, at the *beginning* of the Line B,) represent 100; and by consequence 1 in the *middle* of the Line B will be 1000, and for the *same* reason 10 at the end of the said Line (which is also the *end* of the Rod) will be 10000.

But keeping the Rule as it now stands, let 10 at the end of the fourth Radius (*viz.* at B.) represent 1, then shall each Prime in the said fourth Radius represent .1, in the third Radius .01, in the second Radius .001, and in the first .0001 part of an Unit.

So will 2 in the first Radius be .0002, in the second .002, in the third .02, and in the fourth .2 parts.

The *Numbers* and *Divisions* on the Lines being thus explain'd, it will not be difficult to find the Point upon the Line, where any Number given is represented.

As for Example; Suppose the Numbers were 1894 for the first Figure thereof (*viz.* 1.) I count 1 at the beginning of the Line D, for the second Figure I count 8 tenths next following (that is 8 of the greater Divisions betwixt 1 and 2.) then from this Point forward I count 9 *Centesms* for the third Figure, and

for the last Figure 5, I count half the next *Centefm*; so I find the Point *a g*, will represent 1895, and by the same Rule the Number 1715 will be found at the Point *w g*; hence observe,

1. That on a Line of this length, only the *four* first Figures of any Number propos'd can be discover'd, for if the Number given were 189582, it would be represented at the same Point where the former 1895 was found (*viz.*) at *a g*.

2. That all Numbers which after the first Figure have nothing but *Cyphers* (as 20, 200, 2000, &c.) are all represented at the same Point.

So the Numbers 20, 200, 2000, are all represented by the Figure 2, at the beginning of the second *Prime*.

3. All Numbers consisting of *three* Figures, and having a *Cypher* in the *middle*, are found within the first tenth of that *Prime*, at which the first Figure of the Number given is found.

Example, Let the Number given be 308, for the first Figure I count 3 on the Line (which I find at the beginning of the third *Prime*) now there being a *Cypher* in the *second* place, I must not count any of the tenths, but for the last Figure 8, I count 8 *Centesms*, and that is the Point which represents 308.

4. All Numbers consisting of *four* places, and having *two* *Cyphers* in the *middle*, must be sought between the beginning of the *Prime* unto which they belong, and the first *Centefm* of the same *Prime*; so 4005 being given, the first Figure (*viz.*) 4, is found at the beginning of the fourth *Prime*: Now there being *Cyphers* in the second and third places, I must not count any of the tenths or *Centesms*; but for the last Figure 5, I estimate 5 *Millions*, which is about the middle of the first *Centefm*, and that is the Point where 4005 is represented.

Note,

Note, *Decimal Fractions* and *mixt Numbers* are discover'd after the same manner as whole Numbers; for if the first Number 1895, were 18, 95, it will be found at the same Point, (*viz.*) at a g; and by what hath been said, it is easie to find what Number is represented at any Point upon the Line, as will appear in the following Questions of Proportion, which I come now to treat.

In *Arithmetick* I shall take notice of two sorts of Proportion, *Arithmetical* and *Geometrical*:

1. *Arithmetical* Proportion is when *divers* Numbers, being compared together, retain amongst themselves equal Differences, as these, 2, 4, 6, 8, &c. And this is either *continued*, as in the Numbers before propounded (which is also called *Arithmetical Progression*, or a *Rank* of Numbers *Arithmetically* proportional;) or *discontinued*, as in these, 2, 4, 8, 16, or the like.

2. *Geometrical* Proportion, is when *divers* Numbers being compared together, differ amongst themselves, according to the same Rate or Reason, as these, 2, 4, 8, 16, &c. for here, as 2 is half 4, so 4 is half 8, and 8 half 16; this is likewise either *continued*, as in those before propounded (which are also called *Geometrical Progression*, or a *Rank* of Numbers *Geometrically* proportional;) or *discontinued*, as in these, 2, 4, 16, 32; for as 4 is double 2, so is 32 double 16, but so is not 16, being compar'd with 4.

PROBLEM I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

Find one of the Numbers given upon the Line B, and set it against the other given Number on the Line A, then find the same Number upon B (which was last counted upon A) and against it you have this third proportional upon A; and against this third upon B, is the fourth upon A; in like manner, against the fourth upon B, you have the fifth on A, &c.

Example, Let it be required to find a third proportional to these two Numbers 2 and 4, which may bear the same proportion to 4, that 4 bears to 2.

Draw out the sliding Rod till 2 upon B stand against 4 upon A, then against 4 upon B is 8, (the third proportional) upon A; and against this third (i.e. 8) upon B, is 16 upon A; which is the fourth proportional: Likewise against 16 upon B, is 32 the fifth upon A, and against 32 upon B, is 64 the sixth proportional: But now proceeding forward, I find that 64 upon B, will reach beyond the end of the Line A, I therefore seek 64 towards the left hand upon B, and against it I find 128 the seventh proportional; and so proceeding further, you may find the eighth to be 256, the ninth 512, &c. Contrariwise, if it were required to find a third proportional to the same Numbers 2 and 4, which may bear the same proportion to 2, that 2 bears to 4.

Set

Set 4 in the second Radius upon A, to 2 upon B then against 2 upon A (towards the left hand) is 1, the third proportional, and against 1 upon A is .5 the fourth upon B, also against this fourth (*viz.*) upon A is .25, the fifth proportional on B, &c.

Multiplication by the Lines.

Problem II.

One Number being given to be multiplied by another, to find the Product.

IN Multiplication either of whole Numbers, mixed or decimal Fractions; the Proportion is:

As 1 is to the Multiplier;

So is the Multiplicand to the Product.

And the Product of any two Numbers shall have many places as there be in both the Numbers given except when the lesser of them do not exceed so many of the first Figures of the Product; then it will have one less.

1. Example: *Let it be required to multiply by 4.*

Say, $[1 : 4 :: 6 : 24]$ which Analogy or Proportion may be read thus; *As 1 is to 4, So is 6 to 24.*

Therefore, Set 1 upon the Line B, to 4 on the Line A, and then against 6 upon B, is 24 upon A which is the Product sought.

Note, The Unit or first term may be taken upon either of the Lines A or B, but the first and last terms must be counted upon one and the same Line.

and the *Second* on the *other* Line, where the *fourth* will also be found.

The Letters A and B may serve to distinguish the Lines.

2. Example: *Let the two Numbers given be 68 and 26, to find the Product.* The Proportion is, $[1 : 26 :: 68 : 1768.]$

Therefore, Set 1 upon B, to 26 upon A, then against 68 upon B, is 1768 on A, which is the Product sought; Or,

Set 1 upon B, to 68 upon A, then against 26 upon B, is 1768 (on A) the Product. Therefore it matters not *which* of the Numbers given be made the *multiplier*; and *note* also, that the Product hath many places as are in both the Numbers given, because the least of them (*viz.* 26) doth exceed so many the first Figures of the Product, according to the rule before given.

3. Example: *Let 68 be multiply'd by 14.* The Proportion is, $1 : 14 :: 68 : 952.$

Therefore, Set 1 upon B, to 14 upon A, then against 68 upon B, is 952 the Product upon A, and here the Product consists of one place *less* than there are in the two Numbers given, because the lesser of them (*viz.* 14) doth *not* exceed so many of the first figures of the Product.

Now that the Product last found is 952, and not 952.0, nor 9.52, will thus appear: Set 1 upon B, against 14 upon A.

And then against	2	}	upon	28	}	upon
	3			42		
	4			56		
	5			70		
	6			84		
	6.8			95.2		
			A is			B

By this 'tis evident, that the Lines are in effect Table of Multiplication, for having set 1 to the Multiplier, against any Multiplicand, you have the Product; so if 2 be multiply'd by 14, the Product is 28; if 3, the Product will be 42; if 4, the Product will be 56; if 5, 70, &c. Hence I conclude that if the Multiplicand had been but 6.8, the Product would have been 95.2, but the Multiplicand being 68, the Product must be 952; for by taking away the prick, the Fractions in each are made whole Numbers.

When of two Numbers given to be multiply'd, one consists of whole Numbers or *mixt*, and the other of Fractions only, make the *whole* or *mixt* number the *Multiplier*, and having set 1 against it, set the Fraction towards the left hand, and against it you have the *Product*.

4. Example: Let the two Numbers be 27.5, and .8.

Set 1 on A, to 27.5 on B, and then against (which being less than 1, I seek towards the left hand) on A, is 22, the Product on B.

And notwithstanding a Number of more than four places cannot be exactly expressed on a Line of this length, yet the Product of any Multiplication may be discover'd to six or seven places at least.

5. Example: Suppose I were to multiply 2482 by
 54. The proportion is,

$$1 : 54 :: 2482 : 134028$$

Therefore set 1 upon A to 54 upon B, and then
 against 2, upon A, is 108 upon B: Now suppose,

$$2 \text{ be } \left\{ \begin{array}{l} 20 \\ 200 \\ 2000 \end{array} \right\} \text{ will be } \left\{ \begin{array}{l} 1080 \\ 10800 \\ 108000 \end{array} \right\}$$

So the Product will have six places, and against
 2482 the Multiplicand, you may discern (upon the
 Line) the four first of them, (*viz.*) 1340; the two
 last may be found by multiplying (in ones mind) the
 two last Figures of the *Multiplicand*, by the two last
 of the *Multiplicator*; for so shall you discover the two
 last Figures of the *Product*, which in this Example
 will be 28, which plac'd behind the four first (*viz.*)
 1340, makes the Product compleat 134028.

I might here add more Examples, but these al-
 ready given may serve, they containing (I think)
 sufficient Directions for all the Variety that can hap-
 pen in Multiplication.

Division by the Lines.

PROBLEM III.

One Number being given to be divided by another, to find the Quotient.

IN Division both of whole Numbers and mixt, the Proportion is,

As the Divisor is to 1,

So is the Dividend to the Quotient.

Which Quotient shall ever consist of so many Figures as the Dividend hath more than the Divisor except when the Divisor does *not* exceed so many of the first Figures of the Dividend; then it shall have one place more.

1. Example: *Let it be required to divide 24 by 4.*
The Proportion is, $4 : 1 :: 24 : 6$.

Therefore, Set 4 upon B, to 1 upon A, and then against 24 upon B, is 6 upon A, which is the Quotient sought.

2. Example: *Let it be required to divide 1768 by 26.* The proportion is, $26 : 1 :: 1768 : 68$.

Therefore, Set 26 upon B, to 1 upon A, and then against 1768 upon B, is 68 (the Quotient) upon A.

3. Example; *Suppose 952 were to be divided by 14.*
The Proportion is, $14 : 1 :: 952 : 68$.

There

Therefore, Set 14 upon A, to 1 upon B, then against 952 upon A, you have 68 upon B, which is the Quotient requir'd.

Observe here, the Dividend hath but one Figure more than the Divisor, yet the Quotient consists of two Figures, because the Divisor doth *not* exceed so many of the first Figures of the Dividend; but in the First and Second Examples the Quotients have but so many Figures as the Dividend hath more than the Divisor, because the Divisor doth *not* exceed so many of the first Figures of the Dividend, according to the general Rule above given.

This shews (in all Cases) how many Figures must be in the Quotient, the Value of the first of which may be found by the Rule given in the Introduction, Pag. 8.

Or it will thus appear, that the Quotient in the last Example is 68, and not 6.8, nor .68: Set 14 upon A, to 1 upon B,

$$\begin{array}{lcl}
 \text{And then} & \left\{ \begin{array}{l} 28 \\ 42 \\ 56 \\ 70 \\ 84 \\ 95.2 \end{array} \right\} & \text{upon} \\
 \text{against} & \left\{ \begin{array}{l} 14 \\ 21 \\ 28 \\ 35 \\ 42 \\ 49 \end{array} \right\} & \text{A is} \\
 & & \left\{ \begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6.8 \end{array} \right\} \text{ upon B.}
 \end{array}$$

This is but the Converse of the Third Example of Multiplication; and as by that it is manifest, that if 2 were multiplied by 14, the Product would be 28; if 3, it would be 42; if 4, 56, &c. So here it is evident, that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, &c. thus by reading on the Proportion from the

Divisor 14: 28, 56, 70, &c. I find at last, that if the Dividend were but 95.2, the Quotient would be 6.8, but the Dividend being 952, the Quotient must be 68; for by taking away the prick, the Fractions in each are made whole Numbers.

By these Examples it is also evident, that at once setting of the Rule we both *Multiply* and *Divide*.

For if 14 be a Multiplier, set 1 on B, against 14 on A; then against any Multiplicand upon B you have the Product upon A; as appear'd by the third Example of Multiplication.

And without moving the Rule, if you suppose 14 to be a Divisor, then against any Dividend upon A you have the Quotient upon B; as in the last Example of Division.

How by *any Divisor* to find a *Multiplier*, was shewn in pag. 13 of the *Introduction*; it may also be perform'd by the *Lines* more readily.

Thus *having* a Divisor, to find a *Multiplier*:

Set the Divisor given upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is the Multiplier upon B. Example; Suppose 25 were a *Divisor* given to find a *Multiplier*.

Set 25 upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is .04 the *Multiplier* sought.

By a *Multiplier* to find a *Divisor*: This is but the Converse of the former; for having .04 a Multiplier upon B, set it against 1 upon A, and then against 1 upon B, is 25 the Divisor, as before.

Problem IV.

To reduce a Vulgar Fraction into a Decimal.

SET the Denominator (of the Vulgar Fraction given) upon A, to the Numerator thereof upon B, and then against 1 towards the left hand upon B, is the Decimal Fraction sought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25: For,

As 4 upon A, is to 1 upon B; So is 1 upon A, to .25 upon B.

Also $\frac{3}{8}$ will be .375; for As 28 upon A, is to .21 upon B; So is 1 upon A, to .75 upon B.

In like manner this Vulgar Fraction (*viz.*) $\frac{582}{960}$ is equal to this Decimal, *viz.* .60625; for, set 960 upon A, to 582 upon B, then against 1 upon A, is .60625 upon B; and so for any other.

Problem V.

A Decimal Fraction being given, to reduce the same into the known Parts of the Integer.

IF the Decimal be part of a Pound Sterling:

Set 1 upon B, to the Number of Shillings, Pence or Farthings contained in a Pound, upon A, then seek the Decimal given upon B, (towards the left hand) and against it you have the Shillings, Pence or Farthings respectively contain'd in the Decimal given.

Example: Suppose .7625 were to be reduced into Shillings.

Set 1 upon B, to 20 (the Number of Shillings in a Pound) upon A, then against .7625 upon B, (towards the left hand) is 15.25, that is 15 s. and .25 parts of a Shilling, which is equal to 3 d.

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then against .7625 toward the left hand upon B, is 183.2 the Pence contained in .7625.

3. But if the Decimal aforesaid were to be reduced into Farthings.

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then against .7625 upon A, is 732; and so many Farthings are contained in .7625.

Again, suppose .7625 were a Decimal of an Ale-Barrel, to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons in an Ale-Barrel) upon B, then against .7625 upon A, is 24.4 that is, 24 Gallons; and .4 tenth parts of a Gallon; to reduce this .4 into Pints; set 1 upon A, to 8 upon B, then against 4 upon A, is 3.2 Pints: So .7625 of an Ale-Barrel is equal to 24 Gallons, 3 Pints and 2 tenth parts of a Pint.

Problem VI.

Three Numbers being given, to find a fourth in a direct Proportion.

THIS is called the *Rule of Three Direct*, and by the Instrument is wrought thus;

Set the first Number given upon B, to the second upon A, and then against the third Number given upon B, is the fourth Number sought.

Ex.

Example: If 8 Quarters of Malt will make 20 Barrels of Strong-Beer, how many Barrels of such Beer will 22 Quarters make?

Set 8 upon B, to 20 upon A, and then against 22 upon B, is 55 upon A; and so many Barrels will 22 Quarters make, and according to this proportion 24 will make 60 Barrels, 28 will make 70, 32 Quarters will make 80 Barrels, &c.

Problem VII.

To three Numbers given, to find a fourth in an inversed Proportion.

This is called the Rule of Three Inverse, in which observe, that if the third Number be greater than the first, then will the fourth be less than the second. And contrariwise, if the third Number be less than the first, the fourth will be greater than the second; and in either Cases the Rule is this:

Set the third Number upon A, the first (being of like Denomination) upon B, and then against the second Number upon A, you have the fourth upon B.

Example: If 8 Men do any piece of Work in 9 Days; In how many Days can 12 Men do the same Work?

Set 12 upon A, to 8 upon B, then against 9 upon A, is 6 upon B, which is the Answer. For 12 Men may do the same Work in 6 Days, that 8 Men will do in 9 Days.

But if the Question had been, *In how many days can fix Men do the same Work?* The Answer will be 12. For,

As 6 upon B; is to 8 upon A, So is 9 upon B, to 12 upon A.

Problem VIII.

Betwixt two Numbers given, to find a Mean Geometrically Proportional.

SET one of the Numbers given upon C, to the same Number upon D, and then against the other given Number upon C, is the Geometrical Mean sought.

Example: Let the Numbers given be 50 and 72, find a Geometrical Mean, &c.

Set 50 upon C, to ⁵⁰ upon D, and then against 72 upon C, is 60 upon D. So 60 is a Geometrical Mean betwixt 50 and 72. Or thus:

Set 72 upon C, to 72 upon D, and then against 50 upon C, is 60 upon D, the Mean as before.

By the Pen thus:

Multiply one Number by the other, then extract the Square-Root of the Product, this Square-Root is the Geometrical Mean between the two Numbers given; thus 72 Multiplied by 50, is 3600, whose Square-Root is 60, the Geometrical Mean between 50 and 72.

Pro-

Problem IX.

find the Square-Root of any Number under 1000000.

THE *Extraction of Roots* is one of the *hardest Lessons in Arithmetick*, yet by the help of this *Instrument* it may be perform'd with *less trouble* than any of the foregoing Problems: For if the Lines *A* and *D*, be apply'd one to another, so as 10 at the end of *D*, be even with 10 at the end of *C*; I say, the Lines thus apply'd are like a Table shewing the Square Root of any Number by Inspection only; for against any Number upon *C*, you have the Square-Root thereof upon *D*, & *contr.*

Note, When the Number given consists of 1, 3, 5, or 7 places of Integers, seek it in the first Radius on the Line *C*, and against it you have the Root requir'd upon *D*.

Example: Let the Number given be 144; I find it on the first Radius on the Line *C*, and against it is 12, the Root sought. In like manner,

against $\left\{ \begin{array}{l} 576 \\ 20736 \\ 1000000 \end{array} \right\}$ in the first Radius upon *C*, is $\left\{ \begin{array}{l} 24 \\ 144 \\ 1000 \end{array} \right\}$ the Root upon *D*.

2. When the Number given consists of 2, 4, 6, 8 Places of Integers, find it in the second Radius on the Line *C*, and against it you have the Root upon *D*.

Ex.

Example: Let 16 be the Number given.

I seek 16 in the second Radius upon C, and again it is 4, the Root upon D. In like manner,

Against $\left\{ \begin{array}{l} 30.25 \\ 2304 \\ 784996 \end{array} \right\}$ in the 2d. Rad. up-
on C, is $\left\{ \begin{array}{l} 5.5 \\ 48 \\ 886 \end{array} \right\}$ the Root upon B.

Problem X.

To find the Cube-Root of any Number
1000.000.000.

PLace the Lines D and E, so as 10 at the end of D, be even with 10 at the end of E, then against any Number upon E, you have the Cube-Root thereof upon D. *Et Contr.*

Note, 1. When the Number given consists of 4, or 7 places of Integers, find it in the first Radius of the Line E, and against it you have the Cube-Root sought. Example: Let the Number given be 3375: I seek this in the first Radius in the Line E, and against it I find 15 upon D, which is the Cube-Root of 3375, and so is 212 the Cube-Root of 9528128.

2. When the Number given consists of 2, 5, or 8 places of Integers, find it in the second Radius upon E, and against it is the Root sought. Example: Suppose 35.937 were propounded, find this in the second Radius upon E, and against it is 3.3, the Cube-Root upon D; in like manner is 275 the Cube-Root of 20796875.

3. Wh

Sect. II. Of a Superficies.

37

3. When the Number given consists of 3, 6, or 9 places of Integers, seek it in the third Radius, &c. and against it is the Cube-Root: Thus against 125 in the third Radius upon E, I find 5 the Cube-Root, and so likewise is 888 the Cube-Root of 700.227.072.

Lastly, to know how many places of Integers must be in the Cube-Root of any Number given.

Put a Point over the place of Units in the Number given, then omitting 2, point every third Figure toward the left hand, then tell how many Points, for so many places of Integers must the Cube-Root consist of.

S E C T. II.

Of a Superficies.

1. **A** Superficies is a Figure incompass'd about with a Line or Lines, and is either Round or Angular.

2. A Round Figure is that which is contained by one Round Line, and is either a Circle (as Fig. 1.) or an Ellipsis (as Fig. 2.)

3. An Angular Figure is that which doth consist of three or more Angles, from the Number of which they are denominat'd, as a Figure of three Angles is call'd a Triangle; of four, a Quadrangle, &c.

4. A Triangle is a Superficies comprehended by three Right Lines (as Fig. 3.) of these there are six Varieties, which I shall forbear to mention, they being all measured by one and the same Rule.

5. A

5. A *Quadrangle* is a Figure comprehended by four Right-Lines, and is either a *Parallelogram* or *Trapezium*.

6. A *Parallelogram* is a Figure whose opposite Sides are parallel, having equal distances from one another in all places, and is either Right or Oblique.

7. A *Right-Angled Parallelogram*, is that whose Angles are all Right, and is either a *Square* (as Fig. 5.) or an *Oblong* (as Fig. 4.)

8. The *Oblique-Angled Parallelogram*, is that whose Angles are all Oblique, and is either a *Rhombus* (as Fig. 7.) or a *Rhomboides* (as Fig. 6.)

A *Trapezium* is a Quadrangular Figure, in which no two Sides are equal and parallel (as Fig. 8.)

10. *Figures* consisting of more Sides than four, are almost innumerable, but are reducible unto two sorts, *Regular* and *Irregular*, either of which are also called *Polygons*.

11. *Regular Polygons*, are such, whose Sides and Angles are equal; they take their Names from the Numbers of their Sides, as that of five Sides is called a *Pentagon* (as Fig. 9.) that of six Sides, an *Hexagon*, &c. Of *Irregular Polygons* 'tis needless to say any thing, they being measured after the same manner as a *Trapezium*.

I should next proceed to shew the Use of the Rule in the measuring of Lines, Superficies, and Solids: But in order to this I must premise,

1. That every *Magnitude* must be measured by some known kind of *Magnitude* that is *Homogeneous* (or like) to it. A *Line* is measured by a *Line*, as one *Lineal Inch*, *Foot* or *Yard*, &c.

A *Superficies* is measured by a *Superficies*, as one *Square Inch* or *Foot*, &c. A *Solid* is measured by a *Solid*, as one *Cubick Inch*, one *Cubick Foot*, &c.

And

And when it is known, A

How many $\left\{ \begin{array}{l} \text{Lineal} \\ \text{Square} \\ \text{Cubical} \end{array} \right\} \begin{array}{l} \text{Inches} \\ \text{or Feet} \\ \text{are con-} \\ \text{tained} \\ \text{in a} \end{array} \left\{ \begin{array}{l} \text{Line,} \\ \text{Superficies,} \\ \text{Solid,} \end{array} \right.$

Then is the Quantity or Content of either of
est kind of Magnitudes said to be known.

2. That the Measures of *Capacity* commonly used
England are of two Sorts, one for wet Commodi-
es, as Beer, Cyder, Wine, &c. the other for dry
ommodities, as Corn, &c.

And, I here take it for granted, that there are
ree several Gallons, whose Contents are allowed
be as followeth,

Ale $\left\{ \begin{array}{l} \text{Wine} \\ \text{Corn} \end{array} \right\} \text{Gallon} \left\{ \begin{array}{l} 282 \\ 231 \\ 272\frac{1}{4} \end{array} \right\} \text{Solid} \\ \text{Inches.}$

Now from hence are derived the following Ta-
es, shewing how many Solid Inches are contained
any of the other Measures respectively contained
them.

A TABLE of Beer-Measure.

Inches.	Pints.	Quarts.	Gallons.	Firkins.
35 $\frac{1}{4}$				
70 $\frac{1}{2}$	2			
282	8	4		
2538	72	36	9	
5076	144	72	18	2 Kilderkins.
10152	288	144	36	4 2 Barrel.

A TABLE of Ale-Measure.

Inches.	Pints.	Quarts.	Gallons.	Firkins.
35 $\frac{1}{4}$				
70 $\frac{1}{2}$	2			
282	8	4		
2256	64	32	8	
4512	128	64	16	2 Kilderkins.
9024	256	128	32	4 2 Barrel.

By an Act of the First of *William and Mary*; 34
 Gallons is the Barrel, both for Beer and Ale, in all
 places, except within the weekly Bills of Morta-
 lity.

A TABLE of Wine-Measure.

Inches.							
28	$\frac{7}{8}$	Pint.					
57	$\frac{3}{4}$	2	Quart.				
231	8	4	Gallon.				
4158	144	72	18	Rundlet.			
4553	504	252	63	$3\frac{1}{2}$	Hoghead.		
9404	672	336	84	$4\frac{1}{2}$	$1\frac{3}{4}$	Tericon.	
9106	1008	504	126	7	2	$1\frac{1}{2}$	Pipe.
8212	2016	1008	252	14	4	3	2 Tun.

This Table shews, that in 1 Tun there are 2 Pipes, Tericons, 4 Hogsbheads, 14 Rundless, 252 Gallons, 008 Quarts, 2016 Pints, and 58212 solid Inches. The Tables for Beer and Ale are like this, and need no Explanation.

SECT.

S E C T. III.

*The Use of the Rule in measuring of Superficial
and first of a Circle.*

TH E Area or Superficial Content of a Circle is found by the Diameter or Circumference; I shall therefore first shew how by either of these to find the other.

Problem I.

*The Diameter, or Circumference of a Circle
either being given, to find the other.*

TH E Circumference of that Circle whose Diameter is Unity (or 1) is 3.1415926536 , but for our purpose 3.141592 , will suffice: Therefore, as 1 is to 3.141592 , so is the Diameter of any Circle to the Circumference: By the Instrument thus:

Set 1 on the Line A, against 3.141592 on the Line B, then against any Diameter on the Line A you have the Circumference on the Line B; and the contrary, thus: Against

These Dia-
meters, $\left\{ \begin{array}{l} 20 \\ 30 \\ 40 \\ 50 \end{array} \right\}$ you have these
Circumferences, $\left\{ \begin{array}{l} 62.831 \\ 94.247 \\ 125.663 \\ 157.079 \end{array} \right\}$

Or, Contrariwise, against

These Circum-
ferences, $\left\{ \begin{array}{l} 20 \\ 30 \\ 40 \\ 50 \end{array} \right\}$ you have these
Diameters, $\left\{ \begin{array}{l} 6.366 \\ 9.549 \\ 12.732 \\ 15.915 \end{array} \right\}$

And so of any other.

Problem II.

The Diameter of any Circle being given, to find
the Area, (or any part thereof) in Inches, or
in Ale or Wine-Gallons.

1. For the whole Area in Inches.

THE Area of a Circle is equal to the Rect., or
Rect. Angle of half the Diameter into half the
Circumference; that is to say, if half the Diameter
multiplied by half the Circumference, the Product
shall be the Area. Thus, when the Diameter is 1, the Circumference
3.141592, the half of this is 1.570796, which
multiplied by half the Diameter (*viz.* .5) the Product
shall be the Area of that Circle, whose Diameter is 1,
(*viz.*) .785398.

The

The Area's of all Circles are in proportion one to another, as the Squares of their Diameters (2. 12. of *Euglid.*)

Therefore, *As the Square of the Diameter of any Circle is to the Area of that Circle;*

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above-mentioned the Diameter is 1, and the Area .785398: Now the Square of 1 being but 1, it must hold: *As 1 is to .785398, So is the Square of the Diameter of any Circle, to the Area thereof.* So .785398 is a fixed Multiplier; and if an Unit with Cyphers be divided by .785398, the Quotient will be 1.27324, a fix'd Divisor; and by either of these fix'd Numbers the Area of any Circle may be found, either by Multiplication or Division. For if the Square of any Diameter be

Multiplied } by { .785398 { Prod. } will be
Divided } { 1.27324 { Quot. }

the Area in Square Inches, Feet or Yards, according as the Diameter was measur'd by Inches, &c.

But with more expedition by the Instrument Thus:

Set 1 (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

The Rule being thus set, the Lines are like a Table of Circles Areas to all Diameters; for against any Diameter upon the Line D, you have the Area thereof upon C.

Example: Let the Diameter be 20.

Set 1 upon D, to .785398 upon C, and then against 20 upon D, is 314.159, the Area required upon C; and as the Rule now stands, I also find, that if

The Diameter is $\left\{ \begin{array}{l} 25 \\ 30 \\ 40 \end{array} \right\}$ the Area is $\left\{ \begin{array}{l} 490.87 \\ 706.85 \\ 1256.63 \end{array} \right\}$ Inches.

Contrariwise, when the

Area is $\left\{ \begin{array}{l} 300 \\ 400 \\ 500 \end{array} \right\}$ the Diameter is $\left\{ \begin{array}{l} 19.54 \\ 22.56 \\ 25.23 \end{array} \right\}$

2. For the Area in Gallons.

The Area in Inches divided by 282 or 231, gives the Area in Ale or Wine-Gallons respectively, and so for any other Measure expressed in the former Tables; but without knowing the Area in Inches, the Area in Gallons may be found thus: Divide 785398

By $\left\{ \begin{array}{l} 282 \\ 231 \end{array} \right\}$ the Quotient $\left\{ \begin{array}{l} .0027851 \\ .0033999 \end{array} \right\}$ A. G. will be W. G.

The Quotients are the Area of a Circle, whose Diameter is 1, in Ale or Wine-Gallons, and are fixt Multipliers for finding the Area of all Circles in either of these Measures; for if the Square of the Diameter of any Circle be multiplied by either of these Numbers, the Product is the Area in Ale or Wine-Gallons respectively. If

If you would effect this by Division, the several Divisors are thus found: Multiply the Divisor for finding the Area in Inches, viz. 1.27324

By $\left\{ \begin{array}{l} 282 \\ 231 \end{array} \right\}$ the Product $\left\{ \begin{array}{l} 359.05 \\ 294.11 \end{array} \right\}$ A. G.
will be $\left\{ \begin{array}{l} 359.05 \\ 294.11 \end{array} \right\}$ W. G.

The Products are the Divisors sought. And the Square of the Diameter of any Circle divided by one of these, quotes the respective Area.

Having thus found the several fixed *Multipliers* and *Divisors* for finding the Area of a Circle in Inches and Gallons, I shall here represent them all together in the following Table.

	Multipliers.	Divisors.
Inches.	.785398	1.27324
Alc. Gallons.	.0027851	359.054
Wine-Gallons.	.0023099	294.118

But the Area of any Circle may be more readily found by the help of certain fixt Numbers, called *Gage-Points*, and these fixt Numbers are the Diameters of those Circles, whose Content at one Inch deep is equal to the respective Gallon to which they belong; thus the *Gage-Point* for the Alc. Gallon is 18.95, which is the Diameter of that Circle, whose Content is 282, the Square Inches in the Alc. Gallon.

The several *Gage-Points*, are the Square-Roots of the Divisors last mentioned, and by the Rule are found at once by the Lines C and D, for (setting the Lines even at the end)

Against $\left\{ \begin{array}{l} 359.05 \\ 294.11 \end{array} \right\}$ are these $\left\{ \begin{array}{l} 18.95 \\ 17.15 \end{array} \right\}$ A. G.
 these Divis. } Gage-Points }
 upon C, } upon D, } W. G.

To which are set the Letters *a. g.* and *w. g.* upon the Rule.

Now by these *Gage-Points* the Area of any Circle may be thus found.

For Ale-Gallons,

Set 18.95 (the *Gage-Point* for Ale-Gallons) upon D, 1 upon C, then against any Diameter upon D, you have the Area upon C.

So if the Diameter were 40 Inches, the Area will be 4.45 Ale-Gallons: And the Rule being thus set, the Lines are in effect a Table of Circles Areas: For likewise find, that if the Diameter

be $\left\{ \begin{array}{l} 45 \\ 50 \\ 52.5 \\ 58.6 \end{array} \right\}$ the Area $\left\{ \begin{array}{l} 5.64 \\ 6.96 \\ 7.67 \\ 9.56 \end{array} \right\}$ will be $\left\{ \begin{array}{l} 5.64 \\ 6.96 \\ 7.67 \\ 9.56 \end{array} \right\}$ A. G.

The like for Wine-Gallons, by the proper *Gage-Point*.

Note, When the Area of any Circle is sought in Ale-Gallons, if the Diameter be more than 18.95, and less than 100, set the *Gage-Point* upon D, to 1, at the beginning of C; then against any Diameter, from the *Gage-Point* to 100, upon D, you have the Area upon C. Thus the Rule being set for Ale-Gallons, you will find that when the

Di-

$$\begin{array}{l} \text{Diamet. is } \left\{ \begin{array}{l} 30 \\ 40 \\ 60 \\ 80 \\ 100 \end{array} \right\} \end{array} \begin{array}{l} \text{the Area} \\ \text{will be} \end{array} \left\{ \begin{array}{l} 2.507 \\ 4.456 \\ 10.026 \\ 17.825 \\ 27.851 \end{array} \right\} \text{ Gallons.}$$

When the Diameter is less than the *Gage-Point*, or more than 100, then set the *Gage-Point* to 1, in the middle upon C; then against these

$$\begin{array}{l} \text{Diameters up-} \\ \text{on D,} \end{array} \left\{ \begin{array}{l} 15 \\ 13 \\ 10 \end{array} \right\} \begin{array}{l} \text{you have} \\ \text{these Area's} \\ \text{upon C,} \end{array} \left\{ \begin{array}{l} .627 \\ .471 \\ .278 \end{array} \right\} \text{ A. G.}$$

And without moving the Rule, if 10 at the beginning of the Line D, be 100, the Area against it will be 27.851: So against this Diameter, 200, the Area is 111.4 Gallons, against 300, it is 250.7, and so on to 600 Inches Diameter, against which you have 1002.6, &c. by this you may further observe, that if the Diameter be increased by Tens, the Area will increase by Hundreds: Thus,

$$\begin{array}{l} \text{the} \\ \text{Diameter} \end{array} \left\{ \begin{array}{l} 10 \\ 100 \\ 60 \\ 600 \end{array} \right\} \text{ gives } \left\{ \begin{array}{l} .27851 \\ 27.851 \\ 10.026 \\ 1002.6 \end{array} \right\} \text{ Gallons.}$$

The like for Wine-Gallons by the proper *Gage-Point*.

Lastly, To find any part of the Area of a Circle, in Ale or Wine-Gallons.

Set the *Gage-Point* to $\frac{1}{2}$, $\frac{1}{3}$, or any other part of 1, then against the Diameter you have the like part of the Area.

Ex

Example: Let it be required to find the third part of the Area of a Circle in Ale-Gallons.

Set the Gage-Point to $\frac{1}{3}$ of 1, viz. .333, then against any Diameter you have $\frac{1}{3}$ of the Area; thus against 100 is 9.283, which is $\frac{1}{3}$ of 27.85, the whole Area.

Understand the like for Wine-Gallons, Ale or Beer-Barrels.

Problem III. Fig. 2.

Two Diameters (c d and e f) of an Ellipsis, being given, to find the Area or Content in Ale-Gallons.

As the Square of the Diameter of a Circle is to the Area of that Circle; so is the Rect-Angle Product of the greater and lesser Diameters of an Ellipsis, to the Area thereof. Therefore multiply the greater Diameter by the lesser Diameter, then that Product multiplied or divided by the fixed Multipliers or Divisors given (in pages 46 and 47) gives the Area in Inches, Gallons or Barrels, according to the Number made use of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical Mean Proportion between the greater and lesser Diameters, for this Means is the Diameter of a Circle, whose Area is equal to the Area of the Ellipsis.

D

Ex-

Ex-

Example. Fig. 2.

Let the greater Diameter cd be 72 Inches, and the lesser ef , 50; by the foregoing Rule, the Geometrical Mean between them will be found to be 60, the Diameter of a Circle equal to the *Ellipsis* and the Area of a Circle, whose Diameter is 60, will be found to be 10.02 Ale-Gallons. But the Area of an *Ellipsis* may be more easily found by the Rule thus;

Set 359.05 upon B, to one of the Diameters (suppose 50) upon A; then against the other Diameter 72 upon B, you have the Area upon A, which in this Example will be 10.02, Ale-Gallons, the Content of this *Ellipsis* at one Inch deep: The like may be done for Wine-Gallons, if instead of 359.05, you take 249.11.

Problem IV.

To find the Area or Content of a Triangular Superficies.

IN all Right-lined Triangles, Multiply half the longest Side by the Perpendicular (which is always the nearest Distance from the Base to the opposite Angle;) or Multiply half the Perpendicular by the whole Base; the Product will be the Area required.

Example. Let Fig. 3. represent a Triangular Back or Cooler, whose longest Side ab , is 260 Inches, and the Perpendicular Line co , 110 Inches; the half of ab is 130, this multiply'd by 110, gives 14300 for the Area in Square Inches, and this divided

æ. III. *Of a Parallelogram.*

ded by 282, quotes 50.7, the Area in Ale-Gallons;
by the Rule at one Operation, thus:

Set 282 upon B, to 130 upon A; then against
10 upon B, is 50.7 upon A, the Content as be-
fore.

To measure Quadrangular Figures.

Problem V.

*any Right-angled Parallelogram, be it Square
or Oblong: The Product of any two Sides,
including one and the same Angle, is equal
to the Area or Content.*

1. Example.

IN Fig. 5. the Sides bi and ik , are equal; sup-
pose each be 138 Inches; this multiply'd by it
self, is 19044, the Area in Square Inches: For the
Area in Gallons by the Rule:

Set 282 upon B, to 138 upon A; then against 138
upon B, is 67.53, the Area in Ale-Gallons at one Inch
square.

2. Example. Of an Oblong (Fig. 4.)

Suppose ns , 130 Inches, and su , 180, the Pro-
duct of these (being multiply'd) is 23400, the Area
in Square Inches: But for Ale-Gallons, set 282 upon B, to
130 upon A; then against 180 upon B, is 82.97
Ale-Gallons, the Area required.

Problem VI.

In any Oblique-Angled Parallelogram (be it a Rhombus or a Rhomboides) Multiply the shortest Distance between the two longest Sides, by one of the said Sides, the Product is the Area in Inches.

1. Example. *Of a Rhombus (Fig. 7.)*

Suppose the Side p 1, were 130 Inches, and the distance q 7, 108, these multiplied, the Product will be 14040, the Area in Inches: But by the Rule thus:

As 282 is to 108, So is 130 to 49.78, the Area in Ale-Gallons at one Inch deep.

2. Example. *Of a Rhomboides (Fig. 6.)*

Let the Side x 2, be 260, and the Distance y 108, the Product of these is 28080, the Area in Inches. By the Rule:

Set 282 upon A, to 260 upon B; then against 108 upon A, is 99.57, the Area in Ale-Gallons.

Problem VII. Figure 8.

To find the Area or Content of a Trapezium.

A Right Line drawn from one of the Acute Angles to the Angle opposite (as the Line ab) will divide the Trapezium into two Triangles, the Area's of which are equal to the whole Trapezium, and may be found by the 4th Problem of this Section: Or thus; Multiply the Sum of the two Perpendiculars (fe and er) by the half of ab , the Product is the Area of the Trapezium. This is so plain, that it needs no Example.

All other irregular Right-lined Figures consisting of more than four Sides, must be divided into Triangles (which will be ever less by two than the Number of Sides) and then the Area's of all those Triangles are equal to the Area of the whole Figure.

Problem VIII.

The sides of any Regular Polygon being given, to find the Area.

IN any Regular Polygon, multiply half the Sum of the Sides by the Perpendicular (or nearest Distance from the Center to one of the Sides) the Product is the Area.

Example: In a Pentagon (or Figure of five equal Sides) as Fig. 9.

Suppose each Side be one Inch, to find the Perpendicular (CA) we have given $AB = .5$, and the Angle at $C = 36^\circ$.

D 3

There-

Therefore,

	Log.
As the Sine of C = 36 d.	Co. Ar. 0 23078
Is to the Side A B = .5	0 69897
So is the Sine of B = 54 d.	9.90799
To the Side C A = .68819	.83770

which is the Perpendicular sought; and this .68819 multiplied by half the Sum of the Sides, viz. 2.5 the Product is 1.72047, the Area of a Pentagon whose Side is one Inch; and by this method I find the Area of all the other Polygons which are expressed in the following Table. In the first Column of which you have the Names of the Polygons; in the second their Area's in Inches; each of which being divided by 282, quotes the Area in Ale-Gallons, which are the Numbers in the third Column; and in the 4th and 5th Columns you have the respective Divisors.

Names

Names of the Polygons.	Multipliers. Being the Area's in Inches and Gallons when the Side is one Inch.		Divisors, Being the Square of the Sides when the Area is one Inch or one Gallon.	
	Inches.	Ale Gall.	Inches.	Ale Gall.
Trigon	.433013	.0015355	2.309401	651.251
Tetragon	1.000000	.0035461	1.000000	282.000
Pentagon	1.720477	.0061010	.581234	163.908
Hexagon	2.598076	.0092130	.384900	108.542
Heptagon	3.633959	.0128864	.275182	77.601
Octagon	4.828427	.0171221	.207107	58.404
Nonagon	6.181824	.0219214	.161764	45.618
Decagon	7.694209	.0272844	.129968	36.651

Now having the Area (both in Inches and Ale-Gallons) of these Polygons when the Side is Unity, the Area of each may be readily found, the Side being given: For,

As the Square of the Side of any Polygon, is to the Area of that Polygon;

So is the Square of the Side of any other like Polygon, to the Area thereof: Therefore,

If the Area of any Polygon (expressed in the Table) be desired, in Inches or Ale-Gallons.

Multiply or divide the Square of the Side by the respective Multiplier or Divisor, the Product or Quotient will be the Area sought.

Example: For the Area in Inches, suppose the Side of a Pentagon be 50 Inches; the Square is
 D 4 2500.

2500. This multiplied by 1.720477, or divided by .581234, gives 4301.2, the Area in Inches.

For the Area in Ale Gallons, If the Square of (viz. 2500) be multiplied by .006101, or divided by 163.908, the Product or Quotient will be 15.252, the Area sought.

To find the Side by the Area given.

Multiply or divide the given Area by the proper Multiplicator or Divisor; the Product or Quotient will be the Square of the Side, whose Square-Root is the Side sought.

Thus the Area of a *Pentagon* is 15.252 Ale-Gallons. This multiplied by 163.908, or divided by .006101, gives 2500, whose Square-Root is 50, the side sought.

But the Lines C and D, will effect this with much more expedition; for at once setting of the Rule you have the Area in (Inches or Ale-Gallons) upon any given Side, or the Side to any Area given.

Example: For the Area in Ale-Gallons, set upon D, to (the Area in Gallons when the Side is 1, viz.) .006101 upon C; then against any Side upon D, you have the Area upon C, and the contrary.

Thus against 50 upon D, is 15.252. the Area in Ale-Gallons; Also against 22 Gallons, an Area upon C, you have 60 the Side upon D, &c.

SECT. IV.

The Use of the Rule in the Mensuration of Solids, and first of a Prism.

A Prism is a Solid contained by several Planes, two of which being opposite, are called the Bases, and these are equal, Parallel alike, and alike situate; but the other Planes are Parallelograms, in which a Right Line may be every where applied from Base to Base: Under this Name Prism is comprehended that Solid of two Circular Bases, usually called a Cylinder.

Problem I.

To find the Solid Content of a Prism.

Multiply the Area of the Base by the Perpendicular Height, the Product is the Content sought.

Example: Let Fig. 11. represent a Brewer's Tun, in the form of a Square Prism, whose Base g, h, k, i , is equal to Fig. 5. each Side being 138 Inches, and the Perpendicular ($s i$) 30 Inches, how many Ale-Gallons or Beer-Barrels will this Tun contain?

Set 282 upon B, to 138 upon A; then against 138 upon B, is 67.534 upon A, the Content in Ale-Gallons at one Inch deep; this multiplied by 30, the whole Depth, gives 2026.02, the whole Content in Ale-Gallons; this divided by 36, quotes 56.283, the Content in Beer-Barrels.

The Fraction .283 (being reduced) is 10 Gallons, and .188 parts. For,

As 1.000 : 36 :: .283 : 10.188.

2. Example. Of a Round Prism or Cylinder.

Let Fig. 12. represent a round Tun, whose Diameter (*no*) at Top is equal to (*sy*) the Diameter at Bottom, each being 120 Inches, and the Altitude (*ex*) 36 Inches: How many Gallons or Beer-Barrels may this Tun contain? First, for Gallons.

Set the Gage-Point (*ag*) to 36, the Tun's Depth upon C, then against 120 the Diameter, upon D, is 1443.6, the Content in Ale-Gallons.

2. For the Content in Beer-Barrels: 1443.6 divided by 36, quotes 40.1, the Content in Barrels.

Suppose I come to this Tun, and find the Liquor 9 Inches deep: How many Gallons are then contained in the Tun?

Set the Gage-Point to 9; then against 120 is 360.95 Gallons.

Problem II.

The Diameter, Depth and Content of any Cylindrical Tun, any two being given to find the Third.

IN this Problem are three Questions, all resolved at once setting the Rule.

1. By the Depth and Content, to find the Diameter.

Example: Suppose the Depth 40 Inches, and the Content 1800 Ale-Gallons. What is the Diameter?

Set 40 the Depth upon C, to the Gage-Point upon D: Then against 1800 the Content upon C, is 127.1 Inches, the Diameter sought.

2. By the Diameter and Content, to find the Depth, without moving the Rule: Say,

*As 127.1 the Diameter,
is to 1800 the Content;
So is the Gage-Point,
to 40 the Depth*

} upon $\left\{ \begin{matrix} D \\ C \\ D \\ C \end{matrix} \right\}$

3. By the Depth and Diameter to find the Content, the Rule standing as before: Say,

*As the Gage-Point,
is to 40 the Depth;
So is 127.1 the Diameter,
to 1800 the Content*

} upon $\left\{ \begin{matrix} D \\ C \\ D \\ C \end{matrix} \right\}$

Note,

Note, As a Circle is the Base of a Cylinder or round Prism; so a Triangle, Quadrangle, or any other plain Superficies, may represent the Base of a Prism; for if there be Planes erected perpendicularly upon the Lines which encompass any such Superficies, they will generate a Solid which may be called a Prism; and the Content of any such Solid is gotten by multiplying the Area of the Base by the Altitude or distance from one Base to another.

SECT. V.

The Use of the Rule in the Mensuration of a Sphere, and its Frustums.

Problem I.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons, both Ale and Wine.

A Sphere is two third Parts of a Cylinder, whose Diameter and Altitude are equal to the Diameter of the Sphere.

But if the Diameter and Altitude of a Cylinder be 1, the Area of the Base, which is also the Solid Content (for 1 doth not multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, viz. .523598, is the Content of a Sphere, whose Diameter is Unity or 1.

More.

Moreover, all Bodies are in proportion one to another, as the Cube of their like Sides. Therefore,

As the Cube of the Diameter of any Sphere, is to the Content of that Sphere;

So is the Cube of the Diameter of any other Sphere, to the Content thereof.

Now if the Diameter be 1, the Cube thereof is but one: Therefore,

As 1 is, to .523598;

So is the Cube of the Diameter of any Sphere to the Content.

Example. Let the Diameter of a Sphere be, 20 Inches, and the Content required in Solid Inches.

By the Lines D and C, on the Rule, the Cube of 20 will appear to be 8000; this multiplied by .523598, is 4188.78, the Content sought.

For the Content in Gallons.

If .523598 be divided

By $\left. \begin{array}{l} 282 \\ 231 \end{array} \right\} \text{ quotes } \left\{ \begin{array}{l} .0018567 \\ .002266 \end{array} \right\} \text{ the Con- } \left\{ \begin{array}{l} \text{A. G.} \\ \text{W. G.} \end{array} \right.$

Now if the Cube of the Diameter of any Sphere be multiplied by either of these Numbers, the Product will be the Content in Ale or Wine-Gallons respectively: Thus, the Cube of 20 the Diameter. (*viz.*) 8000, multiplied by .001856, is 14.848 the Content in Ale-Gallons; and the like may be done for Wine.

But

But the Content of any Sphere may be more readily found by the Instrument. Thus,

Set 1 upon D, to .523598 upon E; then against any Diameter upon D, is the Content upon E: So against 20 (the Diameter above-mentioned) upon D, is 4188.78, the Content in Inches, as before: In like manner, if the Diameter were 30, the Content will be 14137.14, &c.

For Ale-Gallons thus:]

Set 1 upon D, to .001856 upon E; then against any Diameter upon D, is the Content upon E; as against 20 upon D, is 14.84, the Content as before; and without moving the Rule, I find that if the

Dia-
ter be $\left\{ \begin{array}{l} 30 \\ 40 \end{array} \right\}$ the Con-
tent is $\left\{ \begin{array}{l} 50.13 \\ 118.78 \end{array} \right\}$ A. G.]

Pro-

Problem II.

Having the Altitude of the Fruustum of a Globe, together with the Diameter of its Base, to find the Altitude of the other Fruustum; or (which is all one) the Remainder of the Globe's Axis.

The RULE is,

Divide the Square of the Semidiameter of the Fruustum's Base, by the Altitude of either of the Fruustums, the Quotient will be the Altitude of the other Fruustum.

Example. In Fig. 10. let cb , the Altitude of the Fruustum, be 6 Inches, and dce , the Diameter at the Base, 24, the half of this (*viz.* dc) is 12, which squared, is 144, this divided by the Altitude bc , *viz.* 6, the Quotient is 24, equal to ca , the Altitude of the other Fruustum. Note also, that a Geometrical Mean Proportion between ac and cb , is equal to cd , which doubled, is de , the Diameter of the Fruustum's Base.

Pro-

Problem III.

Having the Altitude of the lesser Frustum of a Sphere, and the Diameter of its Base, to find the Content in Inches or Gallons.

Multiply the Square of half the Diameter of the Base by three times the Altitude; and to this Product add the Cube of the Altitude, the Sum multiply'd

By $\left\{ \begin{array}{l} .523598 \\ .0018567 \end{array} \right\}$ Gives the Con- $\left\{ \begin{array}{l} \text{Inches.} \\ \text{tent in Ale-Gall.} \end{array} \right\}$

Example: In Fig. 10. let $d c e$, the Diameter of the lesser *Frustum* ($c d b e$) be 24 Inches, and $c d$ its Altitude, 6 Inches; What is the Content in square Inches?

$c e$ squared =
3 times $b c$ =

144
18

1152
144

Cube of $b c$ =

2592
216

2808

2808 multiplied

Content.

By $\left\{ \begin{array}{l} .52398 \\ .0018567 \end{array} \right\}$ gives $\left\{ \begin{array}{l} 1470.263, \text{ Inc.} \\ 5.2137, \text{ A. G.} \end{array} \right\}$

The

The Content of the lesser *Fruustum* taken from the Content of the whole Sphere, leaves the Content of the greater *Fruustum*.

S E C T. VI.

The Use of the Instrument in the Mensuration of Pyramids and their Frustrums.

A Pyramid is a Body contained under several Planes, set upon one Right-lin'd Base, and meeting in a Point at the Top, which is called the Vertex, also in each of these Planes a Right Line may be every where applied from the Base to the Vertex.

And under this Name *Pyramid* is comprehended that Pyramidical Body (whose Base is a Circle) commonly called a *Cone*.

Problem. I.

To find the Solid Content of a Pyramid.

A Pyramid is one third part of a *Prism*, which hath the same Base and Altitude. Therefore.

Multiply the Area of the Base by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude, in either Cases the Product will be the Content of the Pyramid.

Ex-

1. Example.

In a square Pyramid (as Fig. 17.) suppose the side of the Base $\times p$, be 112 Inches, and the Altitude $\times s$, 90 Inches, what is the Content in Ale-Gallons?

Set 282— } upon 5 to 112 } upon
then against 112 } B } is 44.482 } A,

the Area of the Base in Ale-Gallons. Then set upon A, to 30 upon B, and against 44.482 upon A, is 1334.46, the Content in Ale-Gallons.

2. Example.

There is a round Pyramid or Cone (as Fig. 18.) suppose the Diameter at the Base $\times d$, be 100 Inches, and the Altitude $\times s$, 180; How many Ale-Gallons may this Cone contain?

Set the Gage-Point for an Ale-Gallon, to $\frac{1}{4}$ of the Altitude (viz. 60) upon C: then against 100 upon D, is 1671, the Content required.

Problem II.

To find the Solid Content of the Frustrum of a Pyramid.

BT the Rules foregoing, find the Area of the greater, and also of the lesser Base.

2. Find a Geometrical Mean Proportion betwixt these two Area's.

3. The Sum of these three Numbers being multiplied by one third part of the Altitude, the Product is the Content of the Frustrum sought.

Example. Let $a c e o$. (in Fig. 18.) represent the Frustrum of a round Pyramid or Cone, $a o$ the Diameter of the greater Base is 108 Inches, $c e$, the Diameter of the lesser Base, 81, and $n v$, the Altitude, 6 Inches, to find the Content in Ale-Gallons.

1. Set the Gage-Point to 1 in the middle of the line C; then against 108 upon D, is 32.48, the Area of the greater Base in Ale-Gallons.

2. Set the Gage-Point to 1 at the beginning of the line C; then against 81 upon D, is 18.27, the Area of the lesser Base.

3. Set 32.48 upon C, to 32.48 upon D; then against 18.27 upon C, is 24.36, the Geometrical Mean between the two Area's.

So the Area of the greater Base is _____ 32.48

The Area of the lesser is _____ 18.27

A Geometrical Mean betwixt the 2 Area's is _____ 24.36

The Sum is _____ 75.11

This

This multiplied by $\frac{1}{3}$ of the Altitude (*viz.* 12) gives 901.32, the Content of the Frustum sought.

Or by the Penthus :

1. Square the Diameter of the greater Base.
2. Square the Diameter of the lesser Base.
3. Multiply one of these Diameters by the other. To this Product add the two Squares, and multiply the Sum by the Altitude : The last Product, divided by 1077, quotes the Content in Ale Gallons.

S E C T. VII.

The Use of the Rule in Gauging of Brewer's Tuns

THese Vessels are in several Forms: The most usual may be considered under some of these Varieties.

I. Such whose Bases are equal.

And if the equal Bases be alike (that is, both Round, both Square, &c.) and Parallel, and the Sides of the Tun streight from one Base to the other, these Tuns are called *Prisms*. See Problem I. of this Section.

II. Such

II. Such whose Bases are unequal.

1. If the unequal Bases be alike (suppose both Rectangular Parallelograms, both Elliptical, &c.) parallel, Proportional, and alike situate, and the Sides of the Tun streight, such Tuns are the Frustrums of Pyramids. See Problem II. of this Section.

2. If the unequal Bases be unlike, suppose one round, and the other Elliptical, or if they be alike but not proportional, or if they be both alike and proportional, yet if they be not alike situate, the Tun is called a *Prismoid*, in which 'tis still supposed the Bases are parallel, and Sides of the Tun streight. See Problem VII.

Note, When the Bases are proportional, it will

As the Length of the greater Base, is to the Length of the lesser :

So is the Breadth of the greater Base, to the Breadth of the lesser.

And the Bases are said to be alike situate, when the Length of the greater and lesser Bases are both in the same Plane.

Pro³

Problem I. Fig. 11.

There is a Tun whose Bases are both Equal, Parallel, and Square, each Side of either Base being 138 Inches, and the Depth 33 Inches: what is the Content of this Tun in Ale-Gallons? Answer, 2228.62. For,

SET 282 upon B, to 138 upon A; then against 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole Depth, viz. 33, gives 2228.62, the Content required. Or thus:

Set 16.79 upon D, to 33 (the Tun's Depth) upon C, then against 138 upon D, is 2228.62 upon C, the Content as before.

Note, 16.79, is the Gage-Point for square Vessels, it being the Square-Root of 282, the Inches in an Ale-Gallon.

It is needless to give more Examples of this sort of Tuns; for let the Bases be in any Form whatsoever, if the Tun have the Qualifications above-mention'd, the Content is found by multiplying the Area of the Base by the Depth; and if the Bases are round, as in most Tuns they are, the Content may be found at one Operation, as hath been shewn in the I. and II. Problem of Sect. IV.

Of Tuns, whose Bases are unequal.

Problem II. Fig. 16.

There is a Tun (as a, b, c, d, e, f, g, h,) whose Bases are Rectangular Parallelograms, Unequal but Parallel, and alike situate, and the Sides Proportional (d a) the Length of the greater Base, is 100 Inches, and (d c) the Breadth 80; (h e) the Length of the lesser Base 85, and (h g) the Breadth 68, and the Depth of the Tun 30 Inches: How many Ale-Gallons may this Tun contain?

FOR all Tuns of this sort, this is a general Rule.

Take the Area of the greater Base, the Area of the lesser Base, and a Geometrical Mean Proportional betwixt the two Area's; the Sum of these three multiplied by one third part of the Tun's Depth, gives the Content.

Example either by the Pen, or the Lines A and upon the Rule.

As

As 282 is to $d a$ 100; So is $d e$ 80 to }
 the Area of the greater Base, viz. } 28.36

As 282 is to $b e$ 85; So is $b g$ 68 to the }
 Area of the lesser Base, } 20.49

A Geometrical Mean between these }
 two Area's will be found (the Prob. } 24.11
 VIII. Sect. 1.) to be ————— }

The Sum is ————— 72.97

This multiplied by one third part of the Tun
 Depth, viz. 10, gives 729.76 the Content in Ale
 Gallons.

Problem III. Fig. 16.

There is a Tun whose Bases ($i l k m$ the greater, and $n q o p$ the lesser) are both Elliptical, Parallel, Proportional, and alike situated; the greater and lesser Diameters of each Base being equal to the Lengths and Breadths of the Bases of the former Tun, and the Depth the same; How many Ale-Gallons will this Tun contain?

THis Elliptical Tun may be conceived to be inscribed in the Tun, whose Content was found in the last Problem; all the given Dimensions are the same in both, and the Operation will be the same too; for the first Term in the Proportion we take 359.05 instead of 282. See the Work.

As 359.05 is to ik 100 ; So is lm 80 } 22.281
 to the Area of the greater Base. —

As 359.05 is to no 85 ; So is pq 68 } 16.098
 to the Area of the lesser Base —

The Geometrical Mean between these two
 Area's will be found (by Prob. VIII.
 Sect. I.) to be — 18.938

The Sum is — 56.317

This multiply'd by one third part of the Depth,
 iz . 10, gives 573.17 the Content of this Elliptical
 Tun, in Ale-Gallons.

Problem IV.

There is a Tun (as Fig. 17.) whose Bases are
 both Square, but unequal (xp) the Side of
 the greater Base, is 108 Inches, (ro) the
 Side of the lesser Base 93, and the Depth, (sa)
 30 Inches, what is the Content in Ale-Gal-
 lons ?

THIS Tun being the Frustum of a Pyramid
 may be Gauged by the General Rule in Pro-
 blem II. of this Section. Or thus :

E

2.

1. Square

1. Square the Side of the greater Base, and also the Side of the lesser Base.

2. Multiply the greater Side by the lesser, and to the Product add the two former Squares.

3. The Sum of these three being multiplied by the Depth, gives the Content in Square Inches, which divided by 282, the Quotient is the Content in Gallons.

Example: By the Instrument the Square } 1166
of $x p$ 108 is _____

The Square of $x p$ 93 is _____ 864
108 multiplied by 93, is _____ 10044

Sum is _____ 30312
Multiplied by $\frac{1}{2}$ Altitude _____

The Product, viz. _____ 30312

This divided by 282 quotes 1076.48. the Content sought.

Problem V. Figure 18.

There is a Conical Tun whose Bases are both Circular, (a o) the Diameter of the greater Base is 108 Inches, (c e) the Diameter of the lesser Base 93, and (u n) the depth 30 Inches: What is the Content of this Tun in Ale-Gallons?

THE Dimensions being the same as in Problem IV. of this Section, proceed as is there directed, the last Product will be 303570, this divided by 359.05, quotes 845.48 the Content sought.

Or thus by the Rule.

Subtract the Diameter of the lesser Base from the Diameter of the greater, and add half the Difference to the lesser, the Sum is the Diameter in the middle of the Tun's Depth; which found,

1. Set the Gage-Point (a g) to the Tun's depth upon C.

Then against the Diameter upon D, is a fourth number upon C, which keep.

2. Set the Gage-Point to $\frac{1}{3}$ of the Tuns depth upon C.

Then against $\frac{1}{3}$ the Difference of the Diameters upon D, is a fourth Number, which added to the fourth Number first found, is the Content of the Tun sought.

Example. Figure 18.

c o the Diameter of the greater Base is — 10

c e the Diameter of the lesser Base is — 9

The Difference is — 1

The half Difference is 7.5, this added to 9 makes 100.5 for the Diameter in the middle of the Tuns depth: *Therefore*,

1. Set the *Gage-Point* to 30 the Tuns Depth, the against 100.5 the Diameter is 843.91, which keep.

2. Set the *Gage-Point* to $\frac{1}{3}$ of the Tuns Depth that is, 10; then against 7.5 the Semidifference of the Diameters, is 1.56: This added to 843.91 (the Number first found) the Sum is 845.47; the Content sought; which agrees very well with the former Rule.

Problem VI.

To find the Content of a Pyramidal Tun whose Bases are in the Form of any of the first Regular Polygons, one Side of each Base, and the Tuns Depth being given.

THE Area of each Base of any such Tun may be found by Problem VIII. Section III.

A Geometrical Mean betwixt these Area's may be found by Problem VIII. Section I.

The Sum of the two Area's and the Geometrical Mean between them being multiplied by $\frac{1}{3}$ of the Tuns Depth, gives the Content. Or thus:

By the Rule in the last Problem, square the side of each Base, multiply the greater Side by the lesser; then multiply the Sum of these three by $\frac{1}{3}$ of the Tuns Depth. Lastly, multiply or divide this last Product by the proper Multiplier or Divisor [as Table page 55.] the Product or Quotient will be the Content in Ale Gallons.

Example. Let there be a Pyramidal Tun, whose Bases are both in the form of a Pentagon (or Figure of five equal sides) and let the side of the greater Base be 108, the side of the lesser 93, and the depth 10 Inches; all the same as in the last Problem: Therefore according to the Rule and Example there set down, the last Product will be 303570: This multiplied by .006101, or divided by 163.908, [id. Table, page 55.] gives 1852.08.

To find the Contents of any Tun, whose Parallel Bases be Rectangular Parallelograms, or Ellipses, the sides of the Tun being straight from the Top to the Bottom, whether the Bases be alike or unlike, proportional or proportional, alike situated or inverted, by one general Rule.

E 3 Pro-

Problem VII.

Let Fig. 16. represent a Tun called a Prismoid, the Bases are Rectangular Parallelograms, unequal and disproportional, but parallel and alike situate; (a d) the Length below = 150 Inches, and (a b) the Breadth 100, (e h) the Length above 90, and (e f) the Breadth 70, and the Depth 30: What is the Content in Ale Gallons?

The RULE.

1. **T**O the greater Length (a d) add $\frac{1}{2}$ the lesser Length (e h) and multiply the Sum by the greater Breadth (a b) reserving that Product.
2. To the lesser Length (e h) add $\frac{1}{2}$ the greater Length (a d) and multiply the Sum by the lesser Breadth (e f) add this Product to the former reserved Product.
3. Multiply the Sum of these two Products by of the Tun's Depth, and divide the last Product by 282, the Quotient will be the Content of the Tun in Ale Gallons.

Example. (a d) 150 added to $\frac{1}{2}$ (e h) 45, the Sum is 195; this multiply'd by (a b) 100, the Product is 19500, which keep.

2. (e h) 90 added to $\frac{1}{2}$ (a d) 75, the Sum is 165; this multiplied by (e f) 70, the Product is 11550; this added to the former Product, viz. 19500, the Sum

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um is 31050; this multiplied by $\frac{1}{3}$ of the Depth, viz. 10, the Product is 310500, and this divided by 282, gives 1101.06 the Content in Ale-Gallons.

The truth of this Rule will plainly appear, if we only consider Fig. 13, in which $(abcd)$ is equal to the greater Base, and $(efgb)$ equal to the lesser Base of the Tun in the last Problem (which was represented by Fig. 16.)

In this, Figure 13, $(biou)$ is made equal to $(efgb)$.

Also (ab) 100 less by (ef) 70, is equal to (ai) 30.

And (ad) 150 less (eb) 90, is equal to (sd) 60, the Altitude is 30 as before.

The Lines being drawn, the whole Solid is composed of these parts, viz. three Prisms and a Pyramid.

The $\left\{ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \right\}$ Prism is $\left\{ \begin{matrix} b i o u f e b g \\ i a s a b e \\ o v c n b g \end{matrix} \right\}$

The Pyramid is $o s n d b$.

The Content of these four Solids being added together, gives the Content of the whole Solid $b c d e f g b$; for the whole is equal to all its parts taken together.

And the Content of these may be found thus:

1. Multiply (fg) 90 by (fe) 70, the Product is 6300; this multiplied by (bf) the Altitude, viz. 30, gives 189000 the Content of the Prism, $b i o u f e b g$.

2. Multiply (*ai*) 30 by (*as*) 90, the Product is 2700; this multiply'd by $\frac{1}{2}$ of the Altitude, viz 15, is 40500, the Content of the Prism *iasobe*.

3. Multiply (*en*) 70 by (*no*) 60, the Product is 4200; this multiplied by 15, is 63000, the Content of the Prism *ovcnbg*.

4. Multiply (*sd*) 60 by (*so*) 30, the Product is 1800; this multiply'd by $\frac{1}{3}$ of the Altitude, viz 10, gives 18300, the Content of the Pyramid *sondh*.

$$\begin{array}{rcl}
 \text{So the Content of the } \left. \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right\} \text{ Prism is } & \left\{ \begin{array}{l} 189000 \\ 40500 \\ 63000 \end{array} \right. & \\
 \text{Pyramid is } \underline{\hspace{1.5cm}} & \underline{\hspace{1.5cm}} & 18000 \\
 \text{The Sum } \underline{\hspace{1.5cm}} & \underline{\hspace{1.5cm}} & 310500
 \end{array}$$

Is the Content in square Inches, and is exactly the same with the Content found by the Rule given in the last Problem.

The Truth of this Rule being thus demonstrated, we shall shew its further Usefulness in the following Problems.

Prob

Problem VIII. Figure 16.

There is a Tun (called a Cylindroid) whose Bases are both Elliptical, unequal, and disproportionate (that is, there is not the same Proportion betwixt the Axis and Diameter of the greater Base, as is betwixt the Axis and Diameter of the lesser) let the given Dimensions be the same as in the last Problem, viz.

$$\begin{array}{l} n a = 90 : q p = 70 \} \text{Rectangular} \} \text{above} \\ i k = 150 : l m = 100 \} \text{Conjugates} \} \text{below.} \end{array}$$

And the Depth 30 Inches : What is the Content of this Tun in Ale-Gallons ?

For the Content of this Tun, work by the General Rule, and according to the Example given in the last Problem, only instead of 282, divide the last Product by 359.05. So in this Example (the Numbers being the same as in the last) the last Product will be 310500, this divided by 359.05, quotes 864.78, the Content in Ale-Gallons.

Prob.

Problem IX. Figure 15.

There is a Tun whose Bases are Rectangular Parallelograms, and Parallel, but neither equal nor alike situate (for gh , the Breadth above, is posited directly opposite to ad , the Length below) how many Ale-Gallons will this Tun contain?

The Dimensions are,

bg Breadth = 68 : be Length = 85 above.
 da Length = 100 : dc Breadth = 80 below.
 The Depth 30 Inches.

If the Bases of this Tun were alike situate (as is supposed in Problem II. of this Section) the Content would be 729.76 Ale-Gallons, as was there shewn; but the Bases being inverted as abovesaid, the Content will be found to be 735.81 Ale-Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus (bg) 68 added to $\frac{1}{2}$ (da) 50, is 118; this multiplied by (be) 85, the Product is 10030.

Again (da) 100 added to $\frac{1}{2}$ (bg) 34, is 134; this multiplied by (dc) 80, the Product is 10720, the Sum of these two Products is 20750; which multiply'd by $\frac{1}{3}$ of the Depth, viz. 10, gives 207500, the Content in square Inches; this divided by 282, the Quotient is 735.81 the Content in Ale-Gallons.

Or if the last Product be divided by 359.05 the Quotient will be 577.91 the Content of an Elliptical

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tical Tun, whose Axis above is directly opposite to the shorter Diameter below; and such an Elliptical Tun may be conceived to be inscribed in the other, the respective Axis's and Diameters, Lengths, and Breadths, and also the Depth being the same in both.

Problem X. Figure 14.

To find the Content of a Tun whose Bases are unlike, suppose one Square, and the other Oblong; or one Round, and the other Elliptical, but Parallel one to the other, and the sides of the Tun streight.

The Dimensions are,

Breadth. Length.

$$\begin{array}{lcl} ef = 74 : fg = 100 & \left. \begin{array}{l} \text{Rectangular} \\ \text{Conjugates} \end{array} \right\} & \begin{array}{l} \text{above.} \\ \text{below.} \end{array} \\ dc = 100 : cb = 100 & & \\ \text{Depth } 30 \text{ Inches.} & & \end{array}$$

By the general Rule in Problem VII. Work thus:

1. To (ef) 74 add $\frac{1}{2} (dc)$ 50, the Sum is 124, this multiplied by fg 100, is 12400.

2. To (dc) 100 add $\frac{1}{2} (ef)$ 37, the Sum is 137, this multiplied by (cd) 100 is 13700; this added

to the former Product, viz. 124000, makes 261000, and this multiplied by $\frac{1}{3}$ of the Depth, viz. 10,

the Product is 2610000; this divided by 282, gives 9253, the Content of this Tun in Ale-Gallons, sup-

posing the Bases were one Square, and the other Oblong, as the Figure represents them.

But if the Bases were one Round, and the other Elliptical, the last Product, viz. 2610000 must be

divi-

divided by 359.05, and in this case the Content will be 726.91.

By what hath been said, I presume it will not be difficult to find the whole Content of any Tun, whose Bases are parallel, and Sides streight from Top to Bottom.

S E C T. VIII.

Shewing how to find the Content of any Tun from Inch to Inch, or what any Tun will contain upon every Inch of its Depth, which is commonly call'd Inching of a Tun.

Problem I.

Suppose a Tun whose Bases are both Round, or both Square and Parallel one to another, and in the Sides a streight Line may be every where apply'd from Base to Base: Let the Diameter, or Side of the greater Base be 187.2 Inches, the Diameter or Side of the lesser Base 180, and the Depth 12 Inches, then Inch this Tun (be it Round or Square) from the lesser Base downward.

R U L E.

From the Side or Diameter of the greater, subtract the Side or Diameter of the lesser Base, divide the Difference by the Tuns Depth, and call the Quotient (a).

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For the Side or Diameter of the lesser Base, out (*b*).

Multiply the Number (*b*) by the Number (*a*) and all the Product (*c*). Then,

1. To the Square of (*b*) add the Number (*c*), and one third part of the square of (*a*), the Sum of these three will shew the Content of the first Inch of the Tun.

2. Multiply one third part of the Square of (*a*) by 2; to this Product add the Number (*c*); then multiply the Sum by 2, and to this Product add the Square of (*b*); multiply this Sum by 2, this last Product is the Content of the two first Inches of the Tun.

3. Multiply one third part of the square of (*a*) by 3; then add and multiply as in the last, the third and last Product will shew the Content of the three first Inches of the Tun.

4. Having got these three Numbers, subtract the first from the second, and likewise the second from the third; then the first Number, and each of these Differences or Remainders being severally divided by 232 for square Tuns, or by 359.05 for round Tuns, the respective Quotients will be the Content (in Ale-Gallons) of the first, second and third Inches of the Tun, by which a Table may be made for the whole Tun, as in the Example following.

Example of a Square Tun.

The Side of the great Base ————— 187.2

The Side of the lesser Base ————— 180.0

The Remainder is ————— 7.2

This divided by the Tuns Depth, viz. 12. quotes .6 for the Number (*a*).

The

The Number b (which is the side of the lesser Base) is 180 ; this multiply'd by the Number a , viz. .6, gives 108 for the Number c .

1. The Square of b is _____ 32400
 Number c is _____ 108
 $\frac{1}{3}$ of the Square of a is _____ .12

The Sum is the Content of the first }
 Inch of the Tun, in Inches, viz. } 32508.12

2. $\frac{1}{3}$ of the Square of a is _____ .12
 Multiply by _____ 2

Product _____ .24

Add the Number c _____ 108.00

Sum is _____ 108.24

Multiply by _____ 2

Product _____ 216.48

Add the Square of the Number b _____ 32400.00

Sum is _____ 32616.48

Multiply by _____ 2

Product is the Content of the two }
 first Inches of the Tun, in In- } 65232.96
 ches, viz. }

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$\frac{1}{3}$ of the Square of a _____ .12
Multiply by _____ 3

Product _____ .36
Add the Number b _____ 108.00

Sum is _____ 108.36
Multiply by _____ 3

Product _____ 325.08
Add the Square b _____ 32400.00

Sum is _____ 32725.08
Multiply by _____ 3

Product is the Content of the }
three first Inches of the Tun, in } _____ 98175.24
Inches,

Two first Inches _____ 65232.96

First Inch _____ 32508.12

Rest for second Inch _____ 32724.84

Three first Inches _____ 98175.24

Two first Inches _____ 65232.96

Rest for third Inch _____ 32942.28

Now

Now these three Numbers.

$$\text{viz. } \left\{ \begin{array}{l} 32508.12 \\ 32724.84 \\ 32942.28 \end{array} \right\} \begin{array}{l} \text{Divided} \\ \text{by } 282, \\ \text{quotes} \end{array} \left\{ \begin{array}{l} 115.277 \\ 116.045 \\ 116.816 \end{array} \right.$$

For the Content (in Ale-Gallons) upon the first, second, and third Inches of this Tun, supposing the Bases square.

And if the said Numbers be divided by 359.04, the respective Quotients will

$$\text{be } \left\{ \begin{array}{l} 90.539 \\ 91.142 \\ 91.747 \end{array} \right\} \begin{array}{l} \text{the Content of the first, second and} \\ \text{third Inches of the Tun, when the} \\ \text{Bases are Round.} \end{array}$$

Now, by these Numbers the following Table is composed for the Round Tun. Thus,

Take the first Inch from the second, the Remainder or Difference is .603, which set down in the third Column, against the space betwixt the first and second Inches. In like manner take the second Inch from the third, the Remainder is .605, which place in the third Column under the other, then take .603 from .605, there remains .002 for the second Difference, which is always the same, as in the fourth Column, this second Difference added to the second Number in the third Column viz. .605, makes .607, and this added to the Content of the third Inch, viz. 91.747, makes 92.354, the Content of the fourth Inch; and so by a continual Addition of the second Difference to the first, and of their Sum to the Inch last found, the Table is made.

Depib

Depth. Inches.	Content of every Inch. in Ale Gall.	First Differ.	Second Differ.
1	90.539		
2	91.142	.603	.002
3	91.747	.605	.002
4	92.354	.607	.002
5	92.963	.609	.002
6	93.574	.611	.002
7	94.187	.613	.002
8	94.802	.615	.002
9	95.419	.617	.002
10	96.038	.619	.002
11	96.659	.621	.002
12	97.282	.623	
Sum — 1126.766 — Content of the Tun.			

In this Example we have made a Table for the Round Tun, and the Operation would have been the same for the Square Tun, had we taken the three first Inches of *that* instead of *these*.

Note, When we Inch a Tun by this Rule, we suppose it to be Regular, having all the Qualifications express'd in this Problem. But forasmuch as most Tuns have some Irregularity or Unevenness in their Sides, I shall here lay down a more Practical Rule for Inching of Tuns.

Pre-

Problem II.

There is a Tun whose Diameter at the Bottom is 108 Inches, the Diameter at the top 95, and the Depth 30 ; how many Ale-Gallons will this Tun contain upon every Inch of the Depth ?

TO resolve this Question, there must first be known the whole Content of the Tun, and also the Area of the mean Diameter in the middle of every 10 or 12 Inches of the Depth, and these may be thus found.

Take (with some convenient Instrument) a Diameter in the middle of every 10 Inches of the Tun's Depth, and suppose in this Example the first (at 5 Inches from the Top) be 95.5, the second (at 15 Inches from the Top) 100.5, and the third 105.5, these I set down as in the following Table.

This done, we may consider the Tun either as Round or Square, and first to Incb it as a Round.

Set the *Gage Point* (*ag*) to 1 ; then against the several Diameters already found, you have their respective Area's : So the first Area will be 25.40, the second 28.13, the third 31.00 ; these I set down in the Table against their Diameters, and having added them together, the Sum is 84.53, this multiplied by 10 (or which is all one, remove the Prick one place towards the Right-hand) the Product is 845.3, the whole Content of the Tun.

Note,

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Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no use of Multiplication or Division in Inching the Tun, as there will be when the Diameters are taken in the middle of every Foot or half Foot, for here the Area's of the several Diameters do not only shew the mean Inch in the middle of every 10 Inches, but also the whole Content of those 10 Inches, if you remove the Prick but one place more towards the Right-hand. Thus the mean Inch of the last 10 Inches is 31.00, and if the Prick be removed as abovesaid, it is 310, which is the Content of the Tun at 10 Inches deep; therefore reduce the several Contents in the third Column into Barrels, Firkins, and Gallons, Beer-measure, and set them down in the fourth Column, these added together are 23 Barrels, 1 Firkin, 8 Gallons, and 3 tenths of a Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tun's Depth.	Mean Diameters in the middle of every 10 Inch. of the Tuns Depth.	Mean Area's or Contents of every 10 Inch. of the Tuns Depth.	Content of every 10 Inches Beer-Measure.		
			B.	F.	G.
10	95.5	25.40	2	0	2.0
10	100.5	28.13	7	3	2.3
10	105.5	31.00	8	2	4.0
30		84.53	23	1	8.3

By these Numbers thus found it will be easie to make a Table, which shall shew the quantity of Liquor contain'd in the Tun at any depth, for which observe the following Directions.

1. Set

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons, and Parts, in their respective Columns, and at the top of the first Column against the Content set 0, and under it the Tuns Depth in Inches, 1, 2, 3, 4, 5, &c. this done take the first mean Inch, viz. 25.4, and reduce it into Barrels, Firkins, and Gallons, Beer-Measure, it makes 0 : 2 : 7 : 4 : set this upon a scroll of Paper, and subtract it continually from the whole Content till you come at the tenth Inch of the Tuns depth, which done, if you mistake not, the Remainder there will be 16 : 1 : 6 : 3.

2. Take the second mean Inch, viz. 28, 13, this being reduced as the former is 0 : 3 : 1 : 13, subtract this continually from the last Remainder, till you come at 20 Inches deep, and there your Remainder will be 8, 2, 4.

Take the third and last mean Inch, and reduce and subtract it as before, and if your Work be right, there will be no remainder at the last Inch or Bottom of the Tun; now the Table being made, you may cast away the Fractions as useless. The use of this Table is obvious to the meanest Capacity, without any further Explanation :

In.	P.	B.	G.
0	25	18	30
1	22	30	9
2	22	0	2
3	21	14	1
4	20	25	7
5	19	37	3
6	19	0	8
7	18	2	16
8	17	3	31
9	17	0	4
10	16	1	6
11	15	2	17
12	14	3	40
13	14	0	2
14	13	1	7
15	12	2	0
16	11	2	8
17	10	3	7
18	10	0	6
19	9	1	5
20	8	2	4
21	7	3	0
22	6	3	5
23	5	0	1
24	5	0	6
25	4	1	2
26	3	1	7
27	2	2	3
28	1	2	8
29	0	3	4
30	0	0	0

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For if you come to this Tun, and find 2 Inches of the Depth dry, the quantity of Liquor then in the Tun is 22 B. 0 F. 2 G. If 7 Inches be dry, there remain in the Tun but 18, 2, 1, &c. Thus for round Tuns.

Now, suppose the Tun last mention'd had been square, and the Sides at Bottom and Top equal to the Diameters of the last, and the Depth the same; the Diameters or Sides in the middle of every 10 Inches of the Tuns depth will be the same as in the last; by which the several mean Inches, and also the whole Contents of every 10 Inches are found to be in this Table.

Parts Tuns Depth.	Sides of the Tun in the middle of every 10 Inches.	Mean A- rea's or Content of every ten Inches.	Content of every 10 Inches Beer Measure.		
			B.	F.	G.
10	95.5	32.34	8	2	8.4
10	100.5	35.81	9	2	6.1
10	105.5	39.46	10	3	7.6
30		107.61	29	2	4.1

For, set 282 upon B, to 95.5, the first Diameter upon A, then against 95.5 upon B, is 32.34 the first mean Inch, and after the same manner find the second and third, which being reduced as in the last Example, I find the whole Content to be 1076.1 Ale Gallons (or which is the same thing) 29 B, 2 F, 4 G. this being done, you may proceed to Inch this Tun by the same directions which were given in the last Example.

Pro-

Problem III.

To Inch a Tun whose Bases are unlike, viz. the lower Circular, and the upper Elliptical, suppose the Diameter of the Circular Base 100 Inches, the Axis (or longest Diameter) of the Elliptical Base 100, and the shortest 90, and the Tun's Depth 20 Inches.

TO Inch this Tun, I take the cross Diameter in the middle of every 4 Inches of the Tun's depth: Thus at 2 Inches from the Top I find the longest Diameter 100, and the shortest 91, in the middle of the next four Inches (which is 6 Inches from the Top) the longest Diameter is 100, and the shortest 93, and thus I find the rest, which are in this Table is express'd: Now having these Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tun's Depth may be thus found.

Set 359.05 upon B, to the longest Diameter, (viz. 100) upon A, then against the shorter Diameter (viz. 91) upon B, is 25.34, the first mean Inch, or Area in Ale-Gallons; and after the same manner the second and third will be found, as in the following Table.

Depth.	Axis.	Shor- ter Diam.	Mean Inch. in Ale- Gallons.	Content of every 4 Inches of the Tuns Depth.		
4	100	91	25.34	2	3	2.36
4	100	93	25.88	2	3	4.52
4	100	95	26.47	2	3	6.88
4	100	97	27.02	3	0	0.08
4	100	99	27.57	3	0	2.28
0			132.28	14	2	7.12

Now if the Mean Inches be severally multiplied by 4, and these Products reduced into Barrels, Firkins, and Gallons, Beer-measure, they will give the Numbers in the last Column, which added together are 14 Barrels, 2 Firkins, 7 Gallons, and 12 parts of a Gallon.

Set this down for the first Number of your Table, and then reduce the first mean Inch into Barrels, Firkins, and Gallons, and subtract it 4 times from the whole Content and Remainders; do the like with the other mean Inches, and so complete the Table as in the last Example.

Problem I.

To find the Drip or Fall of a Tun, and to make Allowance for the same in Inching the Tun.

Example.

Admit the Tun last mentioned were so placed, that when the Bottom is but just covered on the side, the Liquor is 4 Inches deep on the side
op-

opposite ; How much must be allow'd for the Fall
this Tun.

By the Operation foregoing the Area of the
Diameter in the middle of the 4 last Inches (th
is, at 2 Inches from the bottom) was found to
27.57, this multiplied by 4, is 110.28 Ale-Gallons
the half of this is 55.14, and so much may
allow'd for the Fall of the Tun ; reduce the
55 14 Gallons into Barrels, Firkins, and Gallons
Beer-measure, gives 1 : 2 : 1.14 ; and this take
from 14 : 2 : 7.12, the Content of the Tun wh
it stands level, there remains 13 Barrels, 0 Firkin
5 Gallons, and 98 parts : And so much will
Tun contain, when the Fall is 4 Inches ; theref
set this at the Top of the Table, then reduce
first mean Inch into Barrels, Firkins,
and Gallons, and subtract it four
times from the whole Content, the
remainder will be 10 : 1 : 3 : 62 : Do
the like with the second, third, and
fourth mean Inches, and you will
find the Remainder at 16 Inches dry,
to be 1 : 1.14 : And so much it
takes to cover the Bottom, as was
before observed.

As for such Tuns as are supposed
to be the Frustums of *Spheroids*, *Pa
rabolick Conoids*, *Hyparabolick Conoids*,
or *Parabolick Spindles*, I shall give
Rules for Gauging and Inching of
these in the Appendix, where I shall
also endeavour to explain the nature
of these Figures, and shew whence
they are derived.

0	13	69
1	12	17
2	11	30
3	11	44
4	10	17
5	9	24
6	8	31
7	8	06
8	7	16
9	6	28
10	6	04
11	5	10
12	4	21
13	3	31
14	3	01
15	2	11
16	1	21

Problem V. Figure 23.

Gauge a Copper, and make Allowance for the Crown ; let a, b, c, d, e, a , represent a Copper whose Content is required.

TO find the several Diameters and Depth of the Copper, together with the Altitude of Crown.

Take a small Cord, or Thread, make one end at a , and extend the other to the opposite Side of the Copper at e , where make it fast, or cause the Person to hold it very straight ; then set one End of the Instrument in the Bottom of the Copper, and move it to and fro, till you find the nearest Distance to the Thread (as at o ,) this Distance (bo) the Depth of the Copper, suppose it be 44 Inch-

In like manner set the End of the Rule upon the Top of the Crown at c , and take the nearest Distance to the Thread (as cv ,) suppose it be 36 Inches, this subtracted from (bo) 44, the Remainder 8, is the Altitude of the Crown.

To find (bd) the Diameter of the Bottom of the Crown.

Measure (ac) the Diameter at the Top, admit it be 115 Inches, then hold a Thread so as a Plumb-line at the End thereof may hang just over b ; by this means you may find the Distance ao , and on the other Side es , suppose each be 14.5, add these together, and subtract their Sum (*viz.* 29) from 115, the Remainder 86, is equal to bd , the Diameter at the Bottom of the Crown : The Diameter which touches the Top of the Crown may

be found by the Instrument, suppose it be 91.4
ches.

Now, to find the Content of the Copper for
the Crown upwards (that is, the part *abcie*)
Depth (*vc*) being 36 Inches, you may take a Di-
meter in the middle of every 6 Inches betwixt *v*
c, which suppose to be as in the second Column
the following Table, the Numbers in the third Co-
lumn are the respective Area's in Ale-Gallons, find
by the *Gage-Point* as before; the fourth Column
shews the Content of every 6 Inches in Barrels, Fir-
kins, and Gallons Beer-measure: These added to-
gether are 29 Barrels, 3 Firkins, 2.5 Gallons.

And so much will the Copper contain after
the Crown is cover'd.

Now if the Crown be taken for the Frustum
of a Sphere, the Content (by the third Problem
Section V.) will be found to be 83.3 Gallons.

But the Content may be more readily found,
near the truth: Thus,

The Diameter *bd* was found to be 86,
the Area to this Diameter is 20.6, this Multiplied
by the Crown's Altitude, viz, 4, gives 82.4, the Con-
tent of the Crown, this Subtracted from the Con-
tent of the part or portion *bci d, ab*, the Reman-
der is the quantity of Liquor that will cover
the Crown; the Content of the part *bci d, ab* may
be thus found, *bi* is 91.4, and *bd* is 86, these
added together are 177.4, the half 88.7 may be taken
for a mean Diameter, the Area to this Diameter
is 21.9, this Multiplied by 8, the Altitude, gives
175.2 the Content.

Now Subtract 82.4 (the Content of the Crown)
from 175.2, the Remainder is 92.8, or 2 Bar. 2
2 Gal, and .8 of a Gal. this added to the Content

Sect. VIII. Of Gauging of Coppers. 99

first found, viz. 29 Bar. 3 F. 2.5 Gal. gives 32 B. 1. F. 5.3 Gal. the Content of the whole Copper, as in the following Table.

Parts of the depth.	Diameter.	Mean Inch.	Content of every 6 Inches in		
			B.	F.	G.
6	113.1	35.62	5	3	6.7
6	109.2	33.21	5	2	1.2
6	105.28	30.86	5	0	5.1
6	101.36	28.61	4	3	0.6
6	97.33	26.37	4	1	5.2
6	93.4	24.29	4	0	1.7
36	Sum		29	3	2.5
To cover the Crown			2	2	2.8
Content Copp.			32	1	5.3

The Content being thus found, you may proceed
Inch the Copper by the same Directions which
are given for Inching of Tuns in the last Pro-
blem.

Problem VI.

be Diameter of any Round, or Side of any
Square Mash-Tun, being given, to find
how many Gallons of Mault it will con-
tain at one Inch deep.

THE Corn-Gallon (which is the Measure we
here account by) is supposed to contain 272.25
bick Inches.

But Mr. Dary, in his compleat Gauger, saith, that an indifferent sort of Mault, and of an indifferent Grinding, when three Worts shall have passed through it, will be more compact by about a sixth part; he therefore concludes, that 227 may be a proper Divisor for Square Mash-Tuns, and 388 for Round ones.

1. For Round Mash-Tuns.

The Diameter of that Circle, whose Area is 227, is 17, and this may be called the *Gauge-Point* for Round Mash-Tuns; and to find the Content of any such Mash-Tun :

Set 17 upon D, to 1 upon C; then against any Diameter upon D, you have the Content in Gallons at one Inch deep: Thus if the Diameter be 113, the Area or Content at once Inch deep would be 44.3 that is, 5 Bushels, and 4.3 Gallons.

2. For Square Mash-Tuns.

If a Mash-Tun be Square or Oblong, the Content at one Inch deep may be thus found: Say,
As 227 is to the Length;
So is the Breadth to the Content.

Example. Suppose the Length 112 Inches, and the Breadth 80, how many Gallons will this Tun contain at one Inch deep? Answer, 39.4: For,

Set 227 upon A, to 80 upon B; then against 112 upon A, is 39.4, the Content upon B.

S E C T. IX.

Of Cask Gauging.

BEFORE we can find the Content of a Cask, we must know the Dimensions of it, viz.

The { Bung } Diameter, } in Inches.
 { Head }
 { Length }

And when these are known, the form or shape of the Cask must be considered; for although the Diameters (abovesaid) and the Length of one Cask may be equal to those of another, yet one of these Casks may contain several Gallons more than the other, and therefore the Content of all Casks cannot be found by one and the same Rule.

Now most Writers in Treating on this Subject, have taken it for granted, that every common Cask (as Butt, Pipe, Hoghead, Barrel, and such like;) is in the form of one of these Solids, viz.

1. The middle Frustum of a *Spheroid*.
2. The middle Frustum of a *Parabolical Spindle*.
3. The middle Frustum of two *Parabolical Conoids*, abutting upon one common Base; or,
4. The middle Frustum of two *Cones*, abutting upon one common Base.

Accordingly they have laid down Rules for finding the Content of these Solids, and by those

Rules they suppose the Content of any Cask may be found,

As for instance.

If the Dimensions of a Cask be

Bung } Diameter } $\begin{matrix} 32 \\ 24 \\ 40 \end{matrix}$ } Inches.
Head }
Length —————

By the Rules given by Mr. Dary, and others, the Content of this Cask is taken for

The middle Frustum of	{	a	{	Spheroid —————	}	is	{	Alc-Gall.		
				Parabolical Spindle.				97.455		
				2				{	Parabolic. Conoids	96.505
				Cones —————				89.136		
								87.934		

The Rules are true, as to the first and last of these Solids, viz.

The middle } a Spheroid,
Frustum of } two Cones.

And these may be represented by Figure 21.

A Cask which is the middle Frustum of a Spheroid is shewn by the outermost Line $d, o, a, d, f, c, o, b, o, c, e, d$, the two straight Lines d, e, c , and d, f, e , are the Heads of the Cask, and the Curved Lines d, o, a, o, d , and c, o, o, c , the Staves.

If a Cask (can) be in the form of the middle Frustum of two *Cones*, it may be represented by the eight Lines d, u, a, u, d , and c, u, b, u, c , the Heads d, e, c , and d, f, c , the Bung-Diameter a, b , the Length e, f , are all the same in both these Casks, and yet 'tis evident by the Figure it self, that one will contain more than the other, and by Example above cited, the difference is 10.25, Gallons, for the *Spheroid* contains 95.455 Gallons, the Frustum of two *Cones* but 87.934.

There are very few (if any) Casks whose Diameters and Length are equal to those abovenamed, it will contain more than 97.455 Gallons (the Contents of the *Spheroid*) nor less than 87.934 (the Content of the Frustum of two *Cones*) so that all the Varieties that can happen will fall betwixt these two, the Staves of any Cask differing from these, all lie between the Lines d, o, a , and d, u, a , in Figure 21.

Now for the other two Solids, viz. the middle Frustum of a *Parabolical Spindle*, and the middle Frustum of two *Parabolical Conoids*; according to Authors above-cited, the Content of the first 6.505 Gallons, and the second 89.136.

Whether the Rules they have given for finding the Contents be true or false, I shall examine in Appendix.

But supposing they are true, yet they do not answer the various cases that may happen in common Practice, for in the Example above, the Content of Cask taken, as the middle Frustum of a *Parabolical Spindle*, is but .95 of a Gallon less than the Content of the Frustum of a *Spheroid*, and the Content of the Frustum of two *Parabolical Conoids* is but 1.931 less than the Frustum of two *Cones*.

Now I presume no Man will pretend to distinguish which Notion a Cask must be taken in, whether as the Frustum of a *Spheroid*, or as the Frustum of a *Parabolical Spindle*, when the Contents differ not much as a Gallon.

Moreover, here is above 7 Gallons Difference betwixt the Contents of the Frustum of a *Parabolical Spindle*, and the Frustum of two *Parabolical Conoids*. So that I may modestly affirm, that there are many Casks whose Content cannot be found by any of these Rules; for it must be granted, that a Cask whose Diameters and Length are equal to those in this Example, may contain some certain quantity betwixt 96 Gallons, and 89.

As suppose 93, I say, if this Cask were to be Gauged by any of the Rules above-cited, it will give the Content at least 3 Gallons too much, or 4 Gallons too little.

This premised, I come now to lay down some short Rules, by which the Content of any Cask may be found very near the Truth.

Problem I. Fig. 21.

There is a Cask supposed to be the middle Frustum of a Spheroid, intercepted between two Plains Parallel, cutting the Axis at Right Angles, let $a b$ the Diameter at the Bung be 32 Inches, $d c$ the Diameter at the Head 24, and the Length $e f$, 40, Inches; How many Ale or Wine-Gallons will this Cask contain?

R U L E.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the Difference of the said Squares; the Sum of these multiplied by the Length, and this Product divided

By $\left\{ \begin{array}{l} 1077 \\ 882 \end{array} \right\}$ Quotes the $\left\{ \begin{array}{l} \text{Ale} \\ \text{Content in} \end{array} \right\}$ Wine $\left\{ \begin{array}{l} \\ \text{Gallons.} \end{array} \right\}$

Example.

Square of $a b$ 32 is 1024

Square of $d c$ 24 is 576

Sum is 1600

Half Sum 800

Half the Difference of the Square is 224

Sum is 1024

F 5

Which

Which multiplied by 40 (the Length of the Cask) gives 10496, this Divided by 1077, Quotes 97.455, the Content in Ale-Gallons.

Or divided by 882, the Quotient will be 119.002, the Content in Wine-Gallons.

Note, The Common Rule for Gauging the Frustum of a Spheroid, (by taking twice the Square of the Bung-Diameter, and the Square of the Head-Diameter, &c.) is somewhat a shorter way than this: But this Rule being more agreeable to the Work in the following Problems, I have thought fit to insert it here: The Reader may use which he pleases.

Problem II.

Suppose a Cask (of the same Diameters, and Length with the former) shall have its Staves betwixt the Bung and Head less Curved, so as to leave out about a third part of the space between the Lines $d o a$, (which represent the Spheroid) and $d n a$, as in Figure 22. How many Gallons will this Cask contain?

R U L E.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add three tenth parts of the Difference of the said Squares; the Sum of these Multiplied by the Length, and the last Product divided by 1077, or 882, Quotes the Content in Ale or Wine-Gallons respectively.

Example,

Example. *The Dimensions as above.*

Sum of the Squares is _____ 1600
 Half Sum is _____ 800
 The Difference of the Squares Multipli- }
 ed by .3, is _____ } 134.4
 Sum of these is _____ 2534.4

This multiplied by 40, gives 101376, and this
 Divided by 1077 quotes 94.128, the Content in
 Ale-Gallons, or by 882, the Quotient will be
 14.938, the Content in Wine-Gallons.

Problem III. Figure 19.

There is a Cask whose Staves, between the
 Bung and Head, have but little Curveture,
 lying nearer to the strait Line a u d, than
 to the Curved Line a o d, What is the
 Content of this Cask in Ale or Wine-
 Gallons?

R U L E.

TO the Sum and half Sum of the Squares of the
 Bung and Head Diameters, add one tenth of the
 Difference of the Squares; the Sum of these Mul-
 tiplied by the Length, and this Product divided as
 above directed, quotes the Content in Ale or Wine-
 Gallons respectively.

Example.

Example. (*Dimensions as above.*)

Sum of the Squares is	_____	1600
Half Sum is	_____	800
One tenth of the Difference is	_____	44
Sum of these is	_____	2444

Which Multiplied by 40, gives 97792, this Divided by 1077, the Quotient is 90.800, the Content in Ale-Gallons ; or by 882, quotes 110.875, the Content in Wine-Gallons.

Problem IV. Figure 21.

To find the Content of a Cask in the form of the middle Frustum of two Cones abutting upon one common Base (if any such can be it may be represented by the Strait Line *d u a, a u d.*

R U L E.

From the Sum and half Sum of the Squares of the Bung and Head Diameters, Subtract half the Square of the Difference of the Diameters themselves. Multiply the Remainder by the Length, and Divide the Product as in the last, the Quotient will be the Content in Ale or Wine-Gallons.

Example

Example. (*Dimensions as above.*)

Sum of the Squares is _____ 1600

Half Sum is _____ 800

Sum of these is _____ 2400

Difference of the Diameter is 8, this } 32

Squared is 64, the half is _____ }

The Remainder is _____ 2368

This Multiplied by 40, gives 94920, and this Divided by 1077 quotes 87.934, the Content in Ale-Gallons; or by 882 the Quotient will be 107.619, the Content in Wine-Gallons.

Thus it appears by the four Problems foregoing, if the Dimensions of a Cask

Be { Bung } Diameter _____ { 32 }
 { Head } _____ { 24 } Inches,
 { Length } _____ { 40 }

The Content in each variety is as followeth;

	Ale-Gall.	Diff.
By Problem	I. 97.455	
	II. 94.128	3.327
	III. 90.800	3.328
	IV. 87.934	2.066.

And

And here it may be observed, that the Differences of the Contents thus found are very near equal : So that by these Rules there is not so much room for Error, as by the Rules which several Authors have prescribed.

I have insisted the longer upon this Business, that I might prevent Mistakes, for we cannot be too exact, where a small Error must needs be prejudicial either to the King, or to the Merchant, especially when the Duty upon Brandy is great.

Problem V.

The Dimension of any Cask being given, to find the Content by the Instrument.

TO do this, we must first find a mean Diameter, that is, such a Diameter as shall reduce the Cask to a Cylinder ; for which purpose I have Calculated these three Tables, for the three first varieties of Casks abovementioned.

A T-

Table for Equating the Diameters of the middle Fruustum of a Spheroid.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.67	0.73	0.80	0.87	0.94	1.01	1.08	1.15	1.22	1.29
1	1.36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1.99
2	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
3	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
4	3.46	3.54	3.61	3.68	3.75	3.82	3.89	3.96	4.03	4.10
5	4.17	4.24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4.81
6	4.88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
7	5.60	5.68	5.75	5.82	5.90	5.97	6.04	6.11	6.18	6.25
8	6.32	6.40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
9	7.05	7.12	7.20	7.28	7.35	7.42	7.49	7.56	7.64	7.71

A Table for the Second Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.62	0.68	0.74	0.81	0.87	0.93	0.99	1.06	1.12	1.18
1	1.24	1.30	1.36	1.43	1.49	1.55	1.62	1.68	1.74	1.80
2	1.86	1.93	1.99	2.05	2.12	2.18	2.24	2.30	2.37	2.43
3	2.49	2.55	2.62	2.68	2.75	2.81	2.88	2.94	3.01	3.07
4	3.13	3.20	3.26	3.33	3.39	3.46	3.52	3.59	3.65	3.72
5	3.78	3.85	3.91	3.98	4.04	4.11	4.18	4.24	4.31	4.37
6	4.44	4.50	4.57	4.63	4.70	4.76	4.83	4.90	4.96	5.03
7	5.10	5.16	5.23	5.30	5.36	5.43	5.49	5.56	5.63	5.70
8	5.77	5.84	5.91	5.97	6.03	6.10	6.17	6.24	6.30	6.37
9	6.44	6.51	6.57	6.64	6.71	6.77	6.83	6.90	6.96	7.02

A Table for the Third Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.55	0.60	0.65	0.70	0.76	0.81	0.87	0.93	0.99	1.05
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.55	1.61
3	1.67	1.73	1.78	1.84	1.90	1.95	2.01	2.06	2.12	2.18
4	2.24	2.30	2.35	2.41	2.46	2.52	2.57	2.63	2.69	2.75
5	2.81	2.87	2.93	2.98	3.04	3.10	3.16	3.21	3.27	3.33
6	3.39	3.45	3.51	3.56	3.62	3.68	3.73	3.80	3.86	3.91
7	3.97	4.02	4.08	4.14	4.20	4.26	4.32	4.38	4.44	4.50
8	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98	5.04	5.10
9	5.16	5.22	5.28	5.35	5.41	5.47	5.53	5.59	5.65	5.71
10	5.77	5.83	5.89	5.96	6.02	6.08	6.14	6.20	6.26	6.32

Now by these Tables the mean Diameter of any Cask is thus found.

Subtract the Head Diameter from the Bung Diameter, then seek the Difference in the Table, taking the whole Inches in the first Column, and the Tenths (if any be) at the Head, in the Angle of meeting you have a Number, which being added to the Head Diameter, gives the mean Diameter sought.

Example

Example. Let the Dimensions be as in the first Problem of this Section, the Difference of the Diameter is 8. In the Table for a *Spheroid*, against 8 on the Side, and under 0, I find 5.6, this added to the Head-Diameter (*viz.* 24) the Sum is 29.6, the mean Diameter sought.

This being obtained, the Content of any Cask may be found by the Instrument at one Operation, for which the Proportion is:

*As the Gage-Point is to the Cask's Length;
So is the mean Diameter to the Content.*

Example. The Length 40, and the mean Diameter 29.6 Inches.

Set the *Gage-Point* for Ale Gallons to 40 upon C, then against 29.6 upon D, is 97.455, the Content in Ale Gallons, which is the same as was found by the first Problem of this Section.

Note. The three Tables above are expressed in three Scales, and placed upon one Side of the Rule against the Line of Inches, by which the mean Diameter of any Casks may be readily found.

To find the Ullage of a Cask.

Problem VI.

Let Figure 21 represent a Cask posited with its Axis ef , Parallel to the Horizon, in part empty, the Surface of the Liquor cutting the Bung Diameter in S , suppose the whole Content 97.455 Gallons, the Bung-Diameter ab , 32 Inches, the wet part sb , 24, the dry part sa , 8; How many Gallons are contained in the Cask? And how many will it require to fill it full?

FOR resolving this, and such like Questions, there is a Line of Segments on the Rule, with a Line of Numbers to slide against it, and upon these Lines the Proportion is:

1. As the Bung Diameter upon the Numbers, is to 100 upon the Segments;

So is the wet or dry parts on the Numbers, to a fourth Number on the Segments.

2. As 100 upon C , is to the Cask's whole Content upon A ;

So is the fourth Number last found to the Answer.

Example.

Example. 1. Set 32 the Bung-Diameter on the Numbers, to 100 on the Segments, then against 8, the dry Inches on the Numbers, is 15.5 on the Segments.

2. Set 100 upon B, to 97.455, the Cask's whole Content upon A, then against 15.5 upon B, you have 15.105 upon A, and so many Gallons it will be to fill up the Cask; this subtracted from 97.455, the whole Content, there rests 82.35, the quantity of Liquor in the Cask; which may otherwise be found thus, having set 32 the Bung-Diameter to 100 on the Segments, against 24 the wet Inches on the Numbers, you have 84.5 on the Segments.

As 100 upon B, is to 97.455 upon A;
So is 84.5 upon B, to 82.35 upon A: the same as before.

Note, The Line of Segments on this Rule is made a Spheroid, and is therefore more exact for all gauging Casks than the Table of Segments, which is proper only for a Cylinder.

The following Tables shew the Ullage of two several Casks in Gallons and Quarts: They were made drawing out the Liquor by an exact Wine-Quart, the dry Inches taken by an Instrument proper for that purpose.

		Cask A	
Dry Inches	Ullage	Dry Inches	Ullage
0	0.00	0	0.00
1	0.01	1	0.01
2	0.04	2	0.04
3	0.09	3	0.09
4	0.16	4	0.16
5	0.25	5	0.25
6	0.36	6	0.36
7	0.49	7	0.49
8	0.64	8	0.64
9	0.81	9	0.81
10	1.00	10	1.00
11	1.21	11	1.21
12	1.44	12	1.44
13	1.69	13	1.69
14	1.96	14	1.96
15	2.25	15	2.25
16	2.56	16	2.56
17	2.89	17	2.89
18	3.24	18	3.24
19	3.61	19	3.61
20	4.00	20	4.00

A Table shewing the Ullage of a Cask in Gallons and Quarts, the Length of the Cask is 37.3 Inches, the Bung-Diameter 26.5, the Head-Diameter 25, and the Content 100 Wine Gallons.

<i>Dry Inches.</i>	<i>Gall. & Quarts.</i>	<i>Dry Inches.</i>	<i>Gall. & Quarts.</i>	<i>Dry Inches.</i>	<i>Gall. & Quarts.</i>
0.71	1	6.19	13 2	10.98	32 2
1.10	2	6.33	14 0	11.09	33 0
1.40	3	6.48	14 2	11.20	33 2
1.62	1 0	6.62	15 0	11.31	34 0
1.82	1 1	6.75	15 2	11.42	34 2
2.00	1 2	6.90	16 0	11.53	35 0
2.16	1 3	7.04	16 2	11.64	35 2
2.30	2 0	7.18	17 0	11.75	36 0
2.44	2 1	7.32	17 2	11.86	36 2
2.57	2 2	7.46	18 0	11.97	37 0
2.69	2 3	7.60	18 2	12.08	37 2
2.80	3 0	7.73	19 0	12.19	38 0
2.91	3 1	7.87	19 2	12.30	38 2
3.01	3 2	8.00	20 0	12.41	39 0
3.11	3 3	8.14	20 2	12.52	39 2
3.20	4 0	8.26	21 0	12.63	40 0
3.30	4 1	8.39	21 2	12.74	40 2
3.39	4 2	8.52	22 0	12.85	41 0
3.48	4 3	8.63	22 2	12.96	41 2
3.57	5 0	8.77	23 0	13.07	42 0
3.65	5 1	8.90	23 2	13.18	42 2
3.73	5 2	9.02	24 0	13.28	43 0
3.81	5 3	9.14	24 2	13.39	43 2
3.89	6 0	9.26	25 0	13.40	44 0
4.05	6 2	9.38	25 2	13.60	44 2
4.21	7 0	9.51	26 0	13.70	45 0
4.37	7 2	9.62	26 2	13.81	45 2
4.53	8 0	9.73	27 0	13.92	46 0
4.69	8 2	9.85	27 2	14.02	46 2
4.84	9 0	9.96	28 0	14.12	47 0
4.99	9 2	10.08	28 2	14.23	47 2
5.15	10 0	10.19	29 0	14.33	48 0
5.30	10 2	10.31	29 2	14.44	48 2
5.45	11 0	10.42	30 0	14.54	49 0
5.60	11 2	10.54	30 2	14.65	49 2
5.75	12 0	10.65	31 0	14.75	50 0
5.90	12 2	10.76	31 2		
6.04	12 3	10.87	32 0		

Table shewing the Ullage of a Cask to every half Gallon, the Length of the Cask is 30.8 Inches, the Bung-Diameter 15.6, the Head-Diameter 22.6, and the Content 64 Wine-Gallons.

Dry Inch.	Gall and Quart	Dry Inch.	Gall. and Quart	Dry Inch.	Gall. and Quart
.80	2	5.76	11 2	9.60	22 2
1.25	1 C	5.94	12 0	9.77	23 0
1.60	1 2	6.12	12 2	9.94	23 2
1.91	2 C	6.30	13 0	10.11	24 0
2.17	2 2	6.48	13 2	10.28	24 2
2.40	3 C	6.66	14 0	10.45	25 0
2.63	3 2	6.84	14 2	10.62	25 2
2.85	4 0	7.01	15 0	10.79	26 0
3.07	4 2	7.19	15 2	10.96	26 2
3.28	5 0	7.36	16 0	11.13	27 0
3.49	5 2	7.54	16 2	11.30	27 2
3.69	6 0	7.71	17 0	11.47	28 0
3.89	6 2	7.89	17 2	11.64	28 2
4.08	7 0	8.06	18 0	11.81	29 0
4.27	7 2	8.24	18 2	11.98	29 2
4.46	8 0	8.41	19 0	12.14	30 0
4.65	8 2	8.58	19 2	12.31	30 2
4.84	9 0	8.75	20 0	12.47	31 0
5.03	9 2	8.92	20 2	12.64	31 2
5.21	10 0	9.09	21 0	12.80	32 0
5.40	10 2	9.26	21 2		
5.58	11 0	9.43	22 C		

The

The Casks by which these Tables were made, are a Brandy Punchion, and a Brandy Hoghead, both from *Bourdeaux*: The Tables will serve indifferently for all such Casks, whose Diameters, Length, and Contents are the same with these, and are so plain that they need no Example to shew their use; for against the dry Inches you have the Gallons and Quarts that the Cask wants of being full, and if the Bung-Diameter be more than half dry, enter the Table with the wet Inches instead of the dry, shews the quantity of Liquor in the Cask: The Tables may by Proportion be converted into Lines and put upon a Rule.

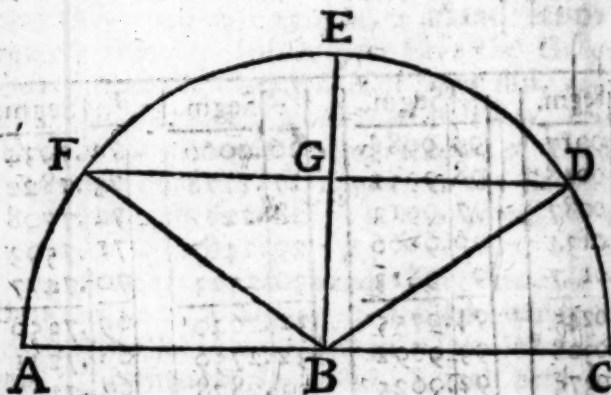
By what hath been already said, 'tis easie (I presume) to find the Ullage of a Cask whose Axis is posited Parallel to the Horizon : But lest this Treatise should be thought imperfect, I shall here insert a Table of the Segments of a Circle, and also their Construction and Use.

A Table of the Segments of a Circle, whose Area is Unity, the Diameter (*viz.* 1.128378) being divided into 100 equal parts.

V. Segm.	S Segm.	V. S Segm.	V. S Segm.
1.0017	99.9983	26.2066	74.7934
2.0048	98.9952	27.2178	73.7822
3.0087	97.9913	28.2292	72.7708
4.0134	96.9866	29.2407	71.7593
5.0187	95.9813	30.2523	70.7477
6.0245	94.9755	31.2640	69.7359
7.0308	93.9692	32.2759	68.7241
8.0375	92.9625	33.2878	67.7122
9.0446	91.9554	34.2998	66.7002
10.0520	90.9480	35.3119	65.6881
11.0598	89.9402	36.3241	64.6759
12.0680	88.9320	37.3364	63.6636
13.0764	87.9236	38.3487	62.6513
14.0851	86.9149	39.3611	61.6389
15.0941	85.9059	40.3735	60.6265
16.1033	84.8967	41.3860	59.6140
17.1127	83.8873	42.3986	58.6014
18.1224	82.8776	43.4112	57.5888
19.1323	81.8677	44.4238	56.5762
20.1424	80.8576	45.4264	55.5636
21.1526	79.8474	46.4491	54.5509
22.1621	78.8369	47.4618	53.5382
23.1737	77.8263	48.4745	52.5255
24.1845	76.8155	49.4873	51.5127
25.1955	75.8045	50.5000	50.5000

The

The Construction of the Table of Segments of
a Circle.



In the Figure here before us, all that part or portion comprehended within the Right Lines DB and BF , and the Circular Line FED , is called the Sector of a Circle, FDG is the Cord of the Segment, GE is the Versed Sine, and GB the Complement thereof, ABC is the Diameter of the Circle, which here we suppose to be 2.

Now, if the Diameter of a Circle be 2, the Length of the Circumference is double the Area of the Circle, for the Circumference is 6.283185 and the Area 3.1415926; hence it evidently follows, that the Length of half the Arch of any Sector of this Circle, is equal to the Area of that Sector, and (as shall be proved anon) the Natural Sine of the said half Arch is equal to the Area of the Triangle in that Sector.

Now, the Area of the Triangle, taken from the Area of the Sector, leaves the Area of the Segment.

Example

Example. Let it be required to find the Area of the Segment FEDG, whose versed Sine GE

1. To find the Area of the Sector.

In order to which we must first know the Quantity of the Angle at the Center, and that is thus discovered.

Seek the Complement of the versed Sine in the Table of Natural Sines, the Degrees and Decimal Parts (or Minutes) thereunto belonging are the Complement of half the Angle at the Center.

Thus BE 1, —GE. 4 = BG. 6, this found in the Table of Natural Sines, gives 36 Degrees and 87

Parts, the Angle ABF, its Complement is 53 d. the Arch FE, which doubled is 106 d. 26'. Arch of the Sector FED, that is, the Quantity of the Angle at the Centers.

Now to find the Length of any Number of Degrees, (that is, the Length of any Arch) of this Circle. Say,

As 360, the Degrees in the whole Circle, to 6.2831852, the Length of the whole Circumference; so is 1, to .01745329, the Length of one Degree.

And this is a common Multiplier for any Number of Degrees, so the Degrees in the Arch FE, 53.13, being Multiplied by .01745329 gives 727787, the Length of the Arch FE, that is, the Area of the Sector BFED, which was required.

2. To find the Area of the Triangle BFD.

The Diameter being .2, I say, half the Natural Sine of the Angle at the Center, when the said Angle does not exceed 90 d. when it does then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus in the Sector BFED, the Angle at the Center is 106.26, its Complement to 180 is 73.74, whose Natural Sine by the Table is .96, the half of this is .48, the Area of the Triangle BFD, which was required.

The truth of which doth thus appear, BF is .1, BG is .6, ergo GB is .8, for $BF^2 + BG^2 = FG^2$ (by 47.1 of Euclid.)

But FG .8 Multiplied by BG .6 gives .48 the Area of the Triangle BFD as before, Then

From the Area of the Sector BFED .9272778
Take the Area of the Triangle BFD .4800000

There remains the Area of the Segment FEDG .4472778

But seeing in all Tables shewing the Area's of the Segments of a Circle, it is supposed the Area of the whole Circle is Unity, the foregoing Table was made by this Proportion.

As the Area of that Circle whose Diameter is 2 viz. 3.1415926, is to the Segment of any part of that Diameter:

So is 1 (the supposed Area of another Circle) to the Segment of the like part of its Diameter. Thus

As 3.1415926, is to .4472778 the Segment last found,

Sect. IX. of the Table of Segments. 123

So is 1 to .14237, the Segment answering to the versed Sine 20, in the Table.

For as the versed Sine of the first Segment, viz. GE was .4 that is $\frac{2}{5}$ of the Radius BE ;

So 20 the versed Sine of this last Segment is $\frac{2}{5}$ of the Radius of the Table.

The Use of the TABLE of Segments,

Is to find the Ullage or Quantity of Liquor remaining in a Cask, whose Axis is posited Parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask, and for this the Rule is ;

To the wet or dry Inches of the Bung-Diameter add a competent number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under the Title VS. gives a Segment, which multiplied by the whole Content of the Cask, the Product shews the quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage if it was the dry.

Example. Let there be a Cask in the form of a Cylinder, whose Bung-Diameter is 33 Inches, the dry part 12, and the Content of the Cask 108 Gallons : how many Gallons are wanting to fill up the Cask ?

12 Divided by 33 quotes 36, which found in the Table under VS gives 3241 ; this Segment multiplied by 108 (the whole Content of the Cask) the Product is 35. and so many Gallons the Cask wants of being full.

Note, If a Cask be in the form of a Cylinder, or near that Figure, the Table will give the Ullage more exactly than the Line of Segments on the Rule :

but if the Bung-Diameter be much greater than the Head-Diameter, the Line of Segments is truer than the Table.

Problem VII. Figure 23.

There is a Cask supposed to be the middle Frustum of a Spheroid, standing upon one Head, with its Axis Perpendicular to the Horizon, the Dimensions

$$\text{Arc } \left\{ \begin{array}{l} \text{E H Length} \text{---} \\ \text{A B Bung-Diameter} \text{---} \\ \text{C D Head-Diameter} \text{---} \end{array} \right. \left\{ \begin{array}{l} 24 \\ 20 \\ 16 \end{array} \right\} \text{Inches.}$$

To calculate any Diameter betwixt the Bung and Head (suppose SV) and by such Diameter to find the Quantity of Liquor in the Cask when part is empty.

I. To find the Diameter the Rule is,

From the Square of the Bung-Diameter (AB) subtract the Square of the Head-Diameter (CD) and divide the Square Root of the Remainder by half the Cask's Length (EO); then multiply this Quotient by the Number of Inches, that the Diameter sought is distant from the Bung, and call this Product your Subducend. Lastly, from the Square of the Bung-Diameter, subtract the Square of your Subducend, the Square Root of the Remainder is the Diameter sought.

Example.

Example. Let it be required to find the Diameter SV, which is supposed to be 6 Inches distant from AB, the Bung-Diameter.

The Square { Bung-Diameter is ————— 400
of the { Head-Diameter is ————— 256

The Remainder is ————— 144

Whole Square-Root is 12, this divided by half the Cask's Length, viz. 12, the Quotient is 1, this multiplied by 6, gives 6 for the Subducend.

Square of the Bung-Diameter ————— 400
Square of the Subducend ————— 36

There remains ————— 364

Which is the Square of the Diameter SV, its Square Root is 19.07878, the Diameter it self; but the Square of it is sufficient for our purpose.

2. To find the Quantity of Liquor in the Cask.

Having found the Square of SV, the Diameter of the Liquor's Surface, the Quantity of Liquor is easily attained:

For by the Rule laid down in the first Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale-Gallons, the half of this is 11.766, for the Content of the lower half, Ant B, and by the Rule now cited, the Content of the part ASVB will be found to be 6.484 Gallons.

G 3.

For

For the Square of AB is	_____	400
The Square of S V is	_____	364
Sum is	_____	764
Half Sum is	_____	382
Half the Difference of the Square is	_____	18
Sum of these is	_____	1164

This multiplied by *eg. viz.* 6, gives 6984, and this divided by 1077 quotes 6.484, the Content of the part ASVB, as above; now this added to the Content of the lower half, *viz.* 11.766, gives 18.25, and so many Gallons are in the Cask: If this be taken from 23.532, the whole Content, there rests 5.282, and so many Gallons are required to fill up the Cask: for this 5.282 Gallons, is the Content of the part SCDV.

If the Cask were less than half full, the Ullage might be found by the same method, and that subtracted from the whole Content leaves the Quantity of Liquor in the Cask.

SECT.

S E C T. X.

Of Gauging of Worts in open Vessels.

FOR this purpose there are two Lines placed on the under side of the two Sliding Pieces of the Rule, which being so plain and obvious as not to need any Description, I shall proceed to shew their Use, which is to find how many Ale-Gallons, and Hundred parts of a Gallon, any small Tub, or such like open Vessel (from 12 to 36 Inches Diameter) will contain at one Inch deep, to perform which observe the following Directions.

 Let the Sliding Piece C D, be pin'd fast at the end towards the Right-Hand, then draw out the other Sliding Piece, till the two ends touch any two opposite sides of the Tub or Vessel, then look what Divisions (both upon the Line of Gallons and Line of Inches upon the Sliding Piece) are cut by the end of the Rule, for that Point doth shew the Diameter of the Tub in Inches and tenth Parts upon one Line; and upon the other, how many Gallons and hundred parts it will contain at one Inch deep.

Thus when the end of the Rule cuts 13.4 on the Line of Inches, it also cuts 0 Gallons & tenths on the Line of Gallons; in like manner, when it cuts 21 on the Inch Line, it will at the same time cut 1.23 on the Line of Gallons; that is, 1 Gallon and .23 hundred parts; and so much will a Tub

of 21 Inches Diameter contains at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half, and then the end of the Rule will cut 1.673 on the Line of Gallons.

Now if the Diameter of the Tub be more than 24.5, pin this Piece at 24.5, and then draw out the other Sliding Piece at the other end, where you will find, that when the end of the Rule cuts 30 Inches, it will also cut 2.51 Gallons; and thus you may draw out this Piece to 36 Inches, where you will find 3 Gallons, 6 tenths and more: *Note*, In all these Cases 'tis the Area only which is sought, the Inches do also shew the Diameter, but this is to make the other more plain.

Now the Tubs or Vessels which Victuallers commonly make use of for cooling their Worts, and working their Ale, are generally Cylindrical or Conical, and in either of these the Bases are Circular or Elliptical.

First of such as are Cylinders, or near that Form.

If the Bases of a Cylindrical Tub be circular, the Rule shews the Area or Content at one Inch deep by Inspection, (as above) this multiplied by the Depth of the Liquor gives the Content.

For instance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 tenths, and the Depth of the Wort 7 Inches and an half:

Multiply

Sect. X. *in open Vessels.* 129

Multiply the Area _____ 1.4
by the Depth, viz. _____ 7.5

70
98

The Product is _____ 10.50

That is, 10 Gallons and an half, which is the Content sought.

But this Content may be found without the Pen, for having the Area and the Depth, you may multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4 the Area upon A; then against 7.5 the Depth upon B, you have 10.5 upon A, the Content as before.

If the Vessel be Elliptical, take the cross Diameters in the middle of the Depth; as suppose the longest Diameter is 60 Inches, and the shortest 40; say,

As 359 is to one of the Diameters;

So is the other Diameter to the Area.

And by this Proportion on the Lines A and B, the Area of the Ellipsis above-mentioned will be found to be 6.68: For,

As 359 upon B, is to 40 upon A;

So is 60 upon B, to 6.68 upon A.

This is the true way of finding the Area of an Ellipsis. But in small Elliptical Tubs you may find the Area with less trouble: Thus,

Find (by drawing out the Rule) the Area of the longest Diameter, and in like manner the Area of the shortest Diameter, half the Sum of these two is the Area sought.

G

For

For Instance.

Suppose the Area of the longest Diameter be	3.62
And the Area of the shortest	2.51
The Sum is	6.13
The half is	3.065

The Area, required, which is very near the truth; for if you work by the former Rule, you will find the true Area to be 3.008.

Secondly, for Conical Vessels.

When the Diameters at Top and Bottom differ not above 4 Inches, you may find the Area in the middle of the Depth, by drawing out the Rule as above directed, and this Area thus found is very near a true mean for the whole Tub.

Example. Let the Diameter at the Bottom of a Conical Tub be 24 Inches, and the Diameter at the Top 28 Inches, in the middle of the Depth of this Tub you will find (by the Rule) the Diameter 26 Inches, and the Area 1.88 Gallons.

And for such Conical Tubs (or Tuns) whose Diameters are above 36 Inches, if they be near a Cylinder, one mean Inch will be sufficient.

But for such as are much Conical and great, you may find the whole Content by Problem V. Sect. VII. And if you desire to Inch any such Tun, observe the Directions in Problem I. Sect. VIII.

There are yet another sort of Vessels which Victuallers do frequently use to work their Ale in, and these are part of a Butt or Pipe set upon one Head, the other being cut off.

The whole Content of these may be found by
Problem VII. Section IX.

I have observed in several Places they cool their
Worts in Brass-Pans, whose Diameter is about 30
Inches, and Depth 10 or 12 Inches; but these are
commonly deeper in the middle by an Inch or two,
than at the Side.

In such Vessels as these, if you take (with the
Rule) the Area of the Liquor's Surface, and multi-
ply it by the Depth taken within 3 or 4 Inches of the
Side, the Product will be the Content very near the
truth.

And for round Bowls, which are like the Segment
of a Globe, if you multiply the Area of the Liquor's
Surface by half the Depth, it gives the Content near
enough in Practice: But in such Cases as these, Ex-
perience will be your best Tutor.

But that I may not be wanting to the meanest
Capacities, I have here inserted a very useful Table,
shewing the Area's of Circles, and Contents of Cy-
linders in Ale-Gallons.

The

T H E
Explanation and Use
O F T H E
TABLE of *Cylinders*.

THE Table consists of two Parts: The first shews the *Area's* of *Circles*, and *Content's* of *Cylinders* from 12 to 42 Inches Diameter, and from 1 to 10 Inches deep, (and by Addition, to any other Depth whatsoever.) The second Part shews only the *Area's* of *Circles* from 42 to 156 Inches Diameter.

In the first Part of the Table, the two first Pages (and so every two next succeeding) contain 10 Columns: In the first of which you have the Diameters of Circles beginning at 12, and continued down (in the said first Column on every left-hand Page) in Inches and Tenths, to 42 Inches.

The other Nine Columns are number'd at the Top, 1, 2, 3, 4, &c. to 9. In the first of these under 1, you have (against their respective Diameters) the *Area's* of *Circles* (or the Content of 1 Inch deep) to the hundred thousandth part of an Ale-Gallon.

In the next Column under 2, is the Content at 2 Inches deep; under 3, the Content at 3 Inches deep; and so on to 9 Inches deep in the last Column under 9.

Thus if the Diameter of a Circle be 29.6 Inches, the Area is 2.4401 Gallons, the Content at 1 Inch deep. And if the Diameter of a Cylinder be 29.6 Inches,

The Content at	Inches Deep is	2	4.88	Gallons.
		3	7.32	
		4	9.76	
		5	12.20	
		6	14.64	
		7	17.08	
		8	19.52	
		9	21.96	

Note, If the former Cylinder of 29.6 Inches Diameter,

Be	Inches Deep, the Contents will be	10	24.4	Gallons.
		20	48.8	
		30	73.2	
		40	97.6	
		50	122.0	
		60	146.4	
		70	170.8	
		80	195.2	
		90	219.6	

For the Content of any Cylinder at 2 Inches, and 20 Inches deep, is express'd by the same Figures; but as 20 is ten times 2, the Area at 2 Inches deep must be made ten times greater, which is done by only putting the Prick, or Point one

one place more towards the Right-hand, From hence the Use of this Table is obvious, as may further appear by the following Example.

There is a Tun in the form of a Cylinder the Diameter is 29.6 and the Depth 27.8 Inches, what is the Content in Gallons?

At 29.6 Inches Diameter,

20	Inches deep is	48.80
7	Inches deep is	17.08
.8	of an Inch deep is	1.95
27.8 Content sought is		67.83

If the mean Diameter of your Tun be above 41 Inches, find it in the first Column of the second part of the Table, for against the whole Inches of your Diameter in the side of the Table, and under the tenths (if any be) you have the Area or Content at one Inch deep; this multiply'd by the whole Depth, gives the Content of the Tun; but this is so plain it needs no Example.

And for Casks, find the mean Diameter by the Direction given in Page 112, and then by the Length and mean Diameter the Content is found as above.

So if the Length of a Cask be 30 Inches, and the mean Diameter 23 Inches, the Content will be found to be 44.2 Gallons, for against 23 the Mean Diameter, and under 3 is 4.42. Now if 3 be 30, 4.42 will be 44.2, the Content sought.

TABLE OF CYLINDERS,

From 12, to 42 Inches Diameter,
and from 1, to 9 Inches Deep:

Very useful in Gauging of
Small Tuns and Casks;

SHOWING

By Inspection, the Content of such
Vessels to the Thousandth
Part of a Gallon, Ale-Mea-
sure.

Diam.in Inches.	D E P T H.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.66
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.93	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.46	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.11
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH

5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.43
2.50	3.00	3.50	4.00	4.50
2.54	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

Diam.in Inches.	D E P T H.			
	1	2	3	4
15.0	0.6266	1.25	1.88	2.50
.1	0.6350	1.27	1.91	2.54
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6604	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH.

5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.55	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

Diam.in Inches.	D E P T H.			
	1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.88	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1020	2.21	3.31	4.41
20.0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86

DEPTH.

5	6	7	8	9
5.51	5.41	6.31	7.22	8.12
5.56	5.47	6.38	7.30	8.21
5.61	5.53	6.45	7.38	8.30
5.66	5.59	6.53	7.46	8.39
5.71	5.66	6.60	7.54	8.49
5.77	5.72	6.67	7.63	8.58
5.82	5.78	6.74	7.71	8.67
5.87	5.85	6.82	7.79	8.77
5.92	5.90	6.89	7.87	8.86
5.97	5.97	6.96	7.95	8.95
6.03	6.03	7.04	8.04	9.05
6.08	6.10	7.11	8.13	9.14
6.13	6.16	7.18	8.21	9.23
6.19	6.22	7.26	8.30	9.34
6.24	6.29	7.34	8.38	9.43
6.30	6.35	7.41	8.47	9.53
6.35	6.43	7.49	8.56	9.63
6.40	6.49	7.57	8.65	9.73
6.46	6.55	7.64	8.74	9.83
6.51	6.62	7.72	8.82	9.93
6.57	6.68	7.80	8.91	10.03
6.63	6.75	7.88	9.00	10.13
6.68	6.82	7.95	9.09	10.23
6.74	6.88	8.03	9.18	10.33
6.80	6.95	8.11	9.27	10.43
6.85	7.02	8.19	9.36	10.53
6.91	7.09	8.27	9.46	10.64
6.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

Diam.in Inches.	D E P T H.			
	1	2	3	4
21.0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.74	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.93	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22.0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.01	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23.0	1.4733	2.95	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.01
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.61	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

DEPTH

5	6	7	8	9
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.68	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.04	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.92
7.24	8.68	10.13	11.58	13.02
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	13.96
7.82	9.39	10.95	12.51	14.08
7.89	9.46	11.04	12.62	14.23
7.95	9.54	11.13	12.72	14.31

Diam.in. Inches.	D E P T H.			
	1	2	3	4
24.0	1.6042	3.21	4.81	6.42
.1	1.6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6.63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25.0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26.0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06

DEPTH.

5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.55
8.16	9.79	11.42	13.05	14.63
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
8.36	10.03	11.70	13.37	15.04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.41	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.64
9.34	11.21	13.08	14.94	16.81
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
0.00	12.00	14.00	16.00	18.00
0.08	12.09	14.11	16.12	18.14

H

Diam.in Inches.	D E P T H.			
	1	2	3	4
27.0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7.27	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96

DEPTH.

5	6	7	8	9
0.15	12.18	14.21	16.24	18.27
0.23	12.27	14.32	16.36	18.41
0.30	12.36	14.42	16.48	18.54
0.38	12.45	14.53	16.61	18.68
0.45	12.55	14.64	16.73	18.82
0.53	12.64	14.74	16.85	18.95
0.61	12.73	14.85	16.97	19.09
0.68	12.82	14.96	17.10	19.23
0.76	12.91	15.06	17.22	19.37
0.84	13.01	15.18	17.34	19.51
0.92	13.10	15.28	17.46	19.65
1.00	13.19	15.39	17.59	19.79
1.08	13.29	15.50	17.71	19.93
1.15	13.38	15.61	17.84	20.07
1.23	13.48	15.72	17.97	20.22
1.31	13.57	15.83	18.10	20.36
1.39	13.67	15.95	18.22	20.50
1.47	13.76	16.06	18.35	20.65
1.55	13.86	16.17	18.48	20.79
1.63	13.96	16.28	18.61	20.93
1.71	14.05	16.39	18.74	21.08
1.79	14.15	16.51	18.86	21.22
1.87	14.25	16.62	18.99	21.37
1.96	14.35	16.74	19.13	21.52
2.04	14.44	16.85	19.26	21.66
2.12	14.54	16.96	19.39	21.81
2.20	14.64	17.08	19.52	21.96
2.28	14.74	17.19	19.65	22.10
2.37	14.84	17.31	19.78	22.26
2.45	14.94	17.43	19.92	22.41

Diam.in Inches.	D E P T H.			
	1	2	3	4
30.0	2.5066	5.01	7.52	10.03
.1	2.5233	5.05	7.57	10.09
.2	2.5401	5.08	7.62	10.16
.3	2.5570	5.11	7.67	10.23
.4	2.5739	5.15	7.72	10.29
.5	2.5908	5.18	7.77	10.36
.6	2.6078	5.21	7.82	10.43
.7	2.6249	5.25	7.87	10.50
.8	2.6420	5.28	7.93	10.57
.9	2.6592	5.32	7.98	10.64
31.0	2.6764	5.35	8.03	10.71
.1	2.6938	5.39	8.08	10.78
.2	2.7111	5.42	8.13	10.84
.3	2.7285	5.46	8.19	10.91
.4	2.7460	5.49	8.24	10.98
.5	2.7635	5.53	8.29	11.05
.6	2.7811	5.56	8.34	11.12
.7	2.7987	5.60	8.40	11.19
.8	2.8163	5.63	8.45	11.27
.9	2.8341	5.67	8.50	11.34
32.0	2.8519	5.70	8.56	11.41
.1	2.8698	5.74	8.61	11.48
.2	2.8877	5.78	8.66	11.55
.3	2.9057	5.81	8.72	11.62
.4	2.9237	5.85	8.77	11.69
.5	2.9418	5.88	8.83	11.77
.6	2.9599	5.92	8.88	11.84
.7	2.9781	5.96	8.93	11.91
.8	2.9963	5.99	8.99	11.98
.9	3.0148	6.03	9.04	12.06

D E P T H.

5	6	7	8	9
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.18
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51
14.26	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13

Diam.in Inches.	D E P T H.			
	1	2	3	4
33.0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.15	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.18	9.26	12.35
.4	3.1069	6.21	9.32	12.43
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6.33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
34.0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13.18
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
35.0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
.5	3.5099	7.02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5605	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

D E P T H.

5	6	7	8	9
15.17	18.20	21.23	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.79
15.53	18.64	21.75	24.85	27.96
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	22.07	26.36	29.66
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	26.14	30.53
17.06	20.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.76
17.75	21.30	24.85	28.39	31.94
17.85	21.42	24.99	28.55	32.13
17.95	21.53	25.12	28.71	32.30

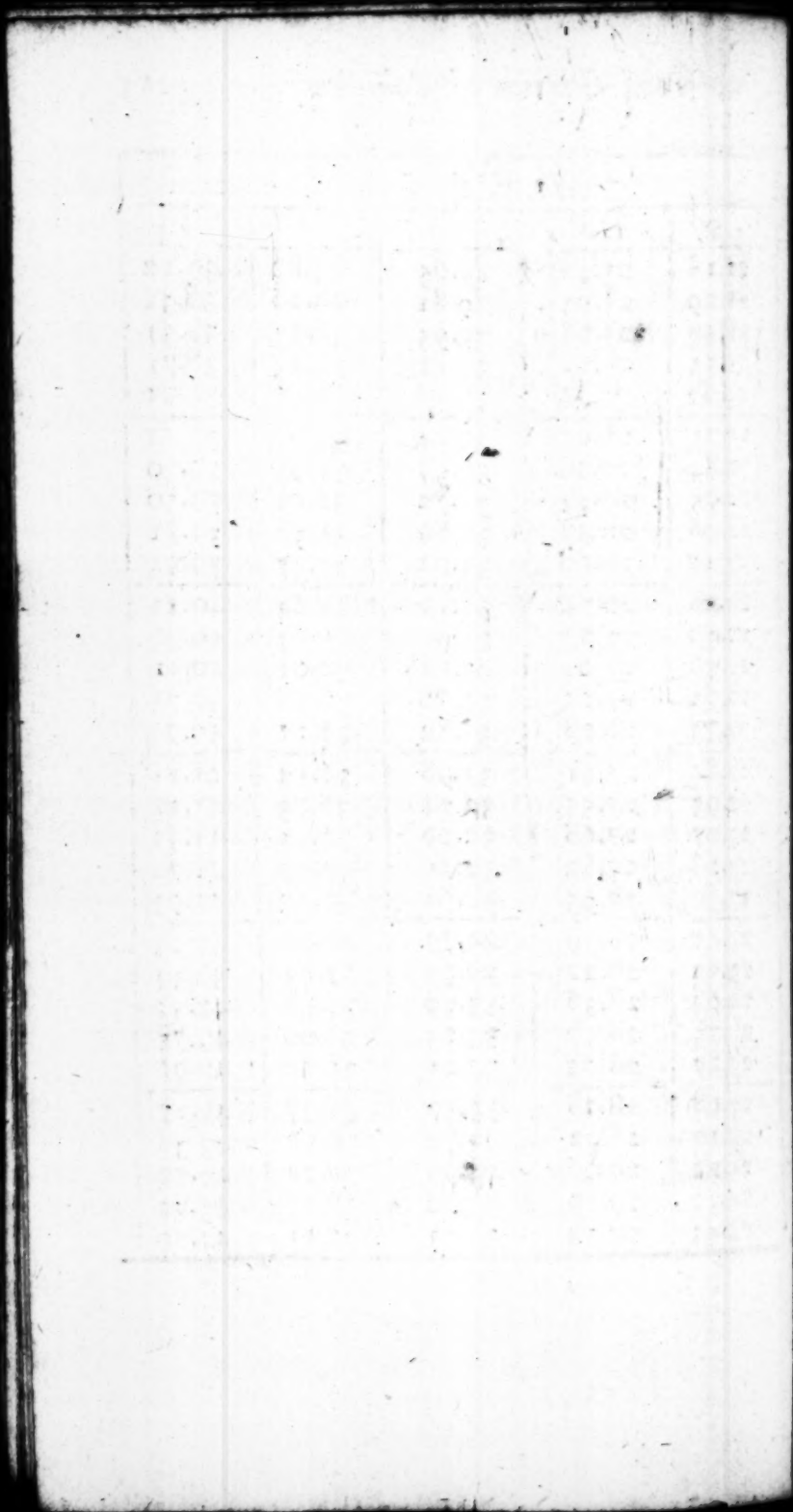
Diam.in Inches.	D E P T H.			
	1	2	3	4
36.0	3.6065	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	7.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

D E P T H.				
5	6	7	8	9
18.05	21.56	25.27	29.87	32.48
18.15	21.78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18.65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.05
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.38	28.44	32.49	36.56
20.43	24.51	28.60	32.68	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93

Diam.in Inches.	D E P T H.			
	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

DEPTH.

5	6	7	8	9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.18	40.71
22.73	27.28	31.82	36.37	40.93
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.63	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	29.34	34.23	39.11	44.00



THE
TABLE

Continued,

From 42, to 156 Inches
DIAMETER,

Shewing the

AREAS

OF

CIRCLES,

TO THE

Ten Thousandth part of a
Gallon, Ale-Measure.

158 *A Table of the Area's of Circles*

Dia.	.0	.1	.2	.3	.4
42	4.9129	4.9363	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4657	5.4904
45	5.6398	5.6649	5.6901	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9961
47	6.1523	6.1735	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0746
51	7.2440	7.2725	7.3010	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9122	7.9419
54	8.1214	8.1515	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5479
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0489	9.0806	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7938	9.8268
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.8098	10.8445
63	11.0541	11.0892	11.1244	11.1596	11.1949
64	11.4078	11.4434	11.4792	11.5150	11.5508
65	11.7670	11.8033	11.8396	11.8759	11.9123
66	11.1319	11.1687	12.2055	11.2424	11.2794
67	12.5023	12.5357	12.5771	12.6145	12.6520
68	12.8723	12.9162	12.9542	12.9922	13.0303
69	13.2599	13.2983	13.3368	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7642	13.8034
71	14.0397	14.0793	14.1189	14.1586	14.1983

	.5	.6	.7	.8	.9
	5.0306	5.0543	5.0780	5.1019	5.1257
	5.2701	5.2944	5.3187	5.3430	5.3675
	5.5152	5.5400	5.5649	5.5898	5.6148
	5.7659	5.7912	5.8167	5.8421	5.8677
	6.0221	6.0480	6.0740	6.1000	6.1261
	6.2839	6.3104	6.3369	6.3635	6.3902
	6.5513	6.5783	6.6054	6.6325	6.6600
	6.8242	6.8518	6.8794	6.9072	6.9349
	7.1027	7.1309	7.1591	7.1873	7.2157
	7.3868	7.4155	7.4443	7.4731	7.5020
	7.6764	7.7057	7.7350	7.7644	7.7939
	7.9717	8.0015	8.0314	8.0613	8.0913
	8.2724	8.3028	8.3333	8.3638	8.3943
	8.5788	8.6097	8.6407	8.6768	8.7029
	8.8907	8.9222	8.9538	8.9854	9.0171
	9.2082	9.2403	9.2734	9.3046	9.3368
	9.5313	9.5639	9.5966	9.6293	9.6621
	9.8600	9.8931	9.9263	9.9596	9.9930
	10.1942	10.2279	10.2617	10.2955	10.3294
	10.5339	10.5682	10.6026	10.6370	10.6714
	10.8793	10.9141	10.9490	10.9840	11.0190
	11.2302	11.2656	11.3011	11.3366	11.3721
	11.5867	11.6227	11.6587	11.6947	11.7309
	11.9488	11.9853	12.0219	12.0585	12.0952
	12.3164	12.3535	12.3906	12.4278	12.4650
	12.6896	12.7272	12.7649	12.8027	12.8405
	13.0684	13.1066	13.1448	13.1831	13.2215
	13.4527	13.4915	13.4915	13.5691	13.6080
	13.8426	13.8819	13.8819	13.9607	13.0002
	14.2381	14.2780	14.2780	14.3579	14.3979

Dia.	.0	.1	.2	.3	.4
72	14.4380	14.4781	14.5183	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9640	15.0049
74	14.2512	15.2925	15.3338	15.3751	15.4165
75	15.6662	15.7080	15.7499	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9586	18.0033
81	18.2730	18.3182	18.3634	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7923	19.8393
85	20.1223	20.1697	20.2172	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7426	21.7907
87	21.0804	21.1289	21.1775	21.2261	21.2747
88	21.5678	21.6169	21.6660	21.7151	21.7643
89	22.0608	22.1104	22.1600	22.2098	22.2595
90	22.5593	22.6095	22.6597	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.2157	23.2666
92	23.5731	23.6244	23.6757	23.7271	23.7785
93	24.0883	24.1402	24.1920	24.2440	24.2960
94	24.6091	24.6615	24.7140	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2945	25.3376
96	25.6675	25.7210	25.7745	25.8282	25.8818
97	26.2050	26.2591	26.3132	26.3673	26.4216
98	26.7481	26.8027	26.8574	26.9121	26.9669
99	27.2968	27.3510	27.4072	27.4625	27.5178
100	27.8510	27.9067	27.9625	28.0184	28.0743
101	28.4108	28.4671	28.5234	28.5798	28.6363

.5	.6	.7	.8	.9
14.6392	14.6796	14.7201	14.7606	14.8012
15.0458	15.0868	15.1278	15.1689	15.2100
15.4580	15.4950	15.5411	15.5827	15.6244
15.8758	15.9178	15.9600	16.0022	16.0444
16.2991	16.3417	16.3844	16.4272	16.4700
16.7280	16.7712	16.8145	16.8578	16.9011
17.1625	17.2062	17.2500	17.2939	17.3378
17.6025	17.6468	17.6912	17.7356	17.7801
18.0481	18.0930	18.1379	18.1829	18.2280
18.4993	18.5448	18.5902	18.6358	18.6814
18.9561	19.0021	19.0481	19.0942	19.1403
19.4184	19.4650	19.5115	19.5582	19.6049
19.8863	19.9334	19.9806	20.0278	20.0750
20.3598	20.4074	20.4551	20.5029	20.5507
20.8388	20.8870	20.9353	20.9836	21.0320
21.3234	21.3722	21.4210	21.4699	21.5188
21.8136	21.8629	21.9123	21.9617	22.0112
22.3093	22.3592	22.4092	22.4592	22.5092
22.8107	22.8611	22.9116	22.9621	23.0128
23.3176	23.3685	23.4196	23.4707	23.5220
23.8300	23.8816	23.9332	23.9848	24.0367
24.3480	24.4001	24.4523	24.5045	24.5568
24.8716	24.9243	24.9770	25.0298	25.0826
25.4008	25.4540	25.5073	25.5606	25.6140
25.9355	25.9893	26.0432	26.0971	26.1510
26.4759	26.5302	26.5846	26.6390	26.6935
27.0127	27.0676	27.1226	27.1776	27.2326
27.5732	27.6286	27.6841	27.7397	27.7953
28.1302	28.1862	28.2423	28.2984	28.3546
28.6928	28.7494	28.8060	28.8627	28.9194

Dia.	.0	.1	.2	.3	.4
102	28.9762	29.0330	29.0899	29.1469	29.2039
103	29.5471	29.6045	29.6620	29.7195	29.7771
104	30.1236	30.1816	30.2396	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2934	31.3525	31.4116	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6661	32.7265
109	33.0898	33.1505	33.2113	33.2722	33.3331
110	33.6997	33.7610	33.8224	33.8838	33.9452
111	34.3152	34.3771	34.4390	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7520	35.8152
114	36.1952	36.2587	36.3223	36.3859	36.4496
115	36.8329	36.8970	36.9612	37.0254	37.0896
116	37.4763	37.5409	37.6056	37.6704	37.7352
117	38.1252	38.1904	38.2557	38.3210	38.3864
118	38.7797	38.8455	38.9113	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.3062	40.3733
121	40.7766	40.8441	40.9116	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3416	42.4103
124	42.8237	42.8928	42.9619	43.0312	43.1004
125	43.5172	43.5868	43.6566	43.7263	43.7961
126	44.2162	44.2865	44.3567	44.4271	44.4974
127	44.9209	44.9916	45.0625	45.1334	45.2043
128	45.6311	45.7024	45.7738	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5627	46.6347
130	47.0682	47.1406	47.2131	47.2857	47.3583
131	47.7951	47.8681	47.9412	48.0143	48.0874

.5	.6	.7	.8	.9
29.2610	29.3181	29.3753	29.4325	29.4898
29.8347	29.8924	29.9501	30.0079	30.0657
30.4140	30.4722	30.5305	30.5889	30.6473
30.9989	31.0577	31.1165	31.1754	31.2344
31.5893	31.6487	31.7081	31.7675	31.8270
32.1853	32.2452	32.3052	32.3652	32.4252
32.7869	32.8474	32.9079	32.9685	32.0291
33.3940	33.4551	33.5161	33.5773	33.6385
34.0068	34.0683	34.1300	34.1917	34.2534
34.6251	34.6872	34.7494	34.8116	34.8739
35.2489	35.3116	35.3744	35.4372	35.5000
35.8784	35.9416	36.0049	36.0683	36.1317
36.5134	36.5772	36.6410	36.7049	36.7689
37.1539	37.2183	37.2827	37.3472	37.4117
37.8001	37.8650	37.9300	37.9950	38.0601
38.4518	38.5173	38.5828	38.6484	38.7140
39.1091	39.1751	39.2412	39.3073	39.3735
39.7719	39.8385	39.9052	39.9719	40.0388
40.4403	40.5075	40.5747	40.6420	40.7093
41.1143	41.1820	41.2498	41.3176	41.3855
41.7939	41.8622	41.9305	41.9989	42.0673
42.4790	42.5475	42.6167	42.6858	42.7547
43.1697	43.2391	43.3086	43.3780	43.4476
43.8660	43.9360	44.0059	44.0760	44.1461
44.5679	44.6384	44.7089	44.7795	44.8502
45.2753	45.3463	45.4174	45.4886	45.5598
45.9883	46.0599	46.1315	46.2032	46.2750
46.7068	46.7790	46.8512	46.9235	46.9958
47.4309	47.5037	47.5764	47.6493	47.7222
48.1606	48.2339	48.3073	48.3806	48.4541

Dia.	.0	.1	.2	.3	.4
132	48.5276	48.6011	48.6747	48.7484	48.8221
133	49.2656	49.3397	49.4139	49.4881	49.5624
134	50.0093	50.0839	50.1586	50.2334	50.3083
135	50.7584	50.8340	50.9099	50.9842	51.0600
136	51.5132	51.5890	51.6650	51.7410	51.8170
137	52.2740	52.3500	52.4262	52.5030	52.5792
138	53.0394	53.1164	53.1933	53.2703	53.3474
139	53.8109	53.8884	53.9659	54.0434	54.1211
140	54.5880	54.6660	54.7440	54.8222	54.9000
141	55.3706	55.4491	55.5268	55.6064	55.6852
142	56.1587	56.2379	56.3170	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1917	57.2716
144	57.7518	57.8321	57.9124	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7993	58.8802
146	59.3672	59.4485	59.5299	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.4291	60.5112
148	61.0048	61.0873	61.1698	61.2524	61.3350
149	61.8320	61.9150	61.9981	62.0812	62.1644
150	62.6648	62.7483	62.8320	62.9157	62.9994
151	63.5030	63.5872	63.6714	63.7556	63.8400
152	64.3470	64.4316	64.5164	64.6012	64.6861
153	65.1964	65.2817	65.3670	65.4523	65.5377
154	66.0514	66.1372	66.2231	66.3090	66.3950
155	66.9120	66.9984	67.0848	67.1713	67.2578
156					

.5	.6	.7	.8	.9
48.8959	48.9697	49.0436	49.1176	49.1916
49.6367	49.7111	49.7856	49.8601	49.9346
50.3832	50.4581	50.5331	50.6082	50.6833
51.1351	51.2110	51.2861	51.3620	51.4374
51.8930	51.9690	52.0450	52.1210	52.1972
52.6560	52.7324	52.8090	52.8860	52.9630
53.4245	53.5017	53.5789	53.6562	53.7335
54.1987	54.2765	54.3543	54.4321	54.5100
54.9786	55.0569	55.1325	55.2136	55.2921
55.7640	55.8428	55.9217	56.0007	56.0797
56.5549	56.6343	56.7138	56.7933	56.8729
57.3515	57.4314	57.5114	57.5915	57.6716
58.1536	58.2341	58.3147	58.3953	58.4760
58.9613	59.0423	59.1235	59.2047	59.2859
59.7745	59.8561	59.9378	60.0196	60.1014
60.5933	60.6755	60.7578	60.8401	60.9224
61.4177	61.5005	61.5833	61.6661	61.7490
62.2477	62.3310	62.4143	62.4978	62.5812
63.0832	63.1671	63.2510	63.3350	63.4190
63.9243	64.0087	64.0932	64.1777	64.2623
64.7710	64.8560	64.9410	65.0261	65.1112
65.6232	65.7088	65.7943	65.8800	65.9657
66.4810	66.5671	66.6533	66.7395	66.8257
67.3444	67.4311	67.5178	67.6045	67.6913

Circles

166 *Circles Area's in Wine Gallons.*

Dia	.0	.1	.2	.3	.4
1	0.0034	0.0041	0.0049	0.0057	0.0067
2	0.0136	0.0150	0.0165	0.0180	0.0196
3	0.0306	0.0327	0.0348	0.0370	0.0393
4	0.0544	0.0572	0.0600	0.0629	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6	0.1224	0.1265	0.1307	0.1349	0.1393
7	0.1666	0.1714	0.1763	0.1812	0.1862
8	0.2176	0.2231	0.2286	0.2342	0.2399
9	0.2754	0.2816	0.2878	0.2941	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4265	0.4341	0.4419
12	0.4896	0.4978	0.5061	0.5144	0.5228
13	0.5746	0.5835	0.5924	0.6014	0.6105
14	0.6664	0.6760	0.6856	0.6953	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8923	0.9033	0.9145
17	0.9826	0.9942	1.0059	1.0176	1.0294
18	1.1016	1.1139	1.1262	1.1386	1.1511
19	1.2274	1.2404	1.2534	1.2665	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5281	1.5425	1.5571
22	1.6456	1.6606	1.6757	1.6908	1.7060
23	1.7986	1.8143	1.8300	1.8458	1.8617
24	1.9584	1.9748	1.9912	2.0077	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3339	2.3517	2.3697
27	2.4786	2.4970	2.5155	2.5340	2.5526
28	2.6656	2.6848	2.7038	2.7230	2.7423
29	2.8594	2.8792	2.8990	2.9189	2.9388
30	3.0600	3.0804	3.1009	3.1215	3.1421

.5	.6	.7	.8	.9
0.0077	0.0087	0.0098	0.0110	0.0123
0.0213	0.0230	0.0248	0.0267	0.0286
0.0417	0.0441	0.0465	0.0491	0.0517
0.0689	0.0719	0.0751	0.0781	0.0816
0.1029	0.1066	0.1105	0.1144	0.1184
0.1437	0.1481	0.1526	0.1572	0.1619
0.1913	0.1964	0.2016	0.2069	0.2122
0.2457	0.2515	0.2573	0.2633	0.2693
0.3069	0.3133	0.3199	0.3265	0.3332
0.3748	0.3820	0.3893	0.3966	0.4040
0.4496	0.4575	0.4654	0.4734	0.4815
0.5312	0.5398	0.5484	0.5571	0.5658
0.6196	0.6289	0.6381	0.6475	0.6569
0.7148	0.7247	0.7347	0.7447	0.7548
0.8168	0.8274	0.8381	0.8488	0.8596
0.9256	0.9369	0.9442	0.9596	0.9711
1.0412	1.0532	1.0652	1.0773	1.0894
1.1636	1.1763	1.1889	1.2017	1.2145
1.2928	1.3061	1.3195	1.3329	1.3464
1.4288	1.4428	1.4569	1.4710	1.4852
1.5716	1.5863	1.6010	1.6158	1.6307
1.7212	1.7366	1.7520	1.7675	1.7830
1.8776	1.8937	1.9097	1.9259	1.9421
2.0408	2.0575	2.0743	2.0911	2.1080
2.2108	2.2282	2.2457	2.2632	2.2808
2.3876	2.4057	2.4238	2.4420	2.4603
2.5712	2.5900	2.6088	2.6277	2.6466
2.7616	2.7811	2.8005	2.8201	2.8396
2.9588	2.9789	2.9991	3.0193	3.0396
3.1628	3.1836	3.2045	3.2254	3.2464

Dia	.0	.1	.2	.3	.4
31	3.2674	3.2885	3.3097	3.3309	3.3523
32	3.4816	3.5034	3.5253	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7702	3.7929
34	3.9304	3.9536	3.9768	4.0001	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4555	4.4801	4.5049
37	4.6546	4.6798	4.7051	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2513	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7713	5.7993	5.8275
42	5.9976	6.0262	6.0549	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6725	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2571	7.2885	7.3201
47	7.5106	7.5426	7.5747	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9318	7.9647
49	8.1634	8.1968	8.2302	8.2637	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9129	8.9477	8.9827
52	9.1936	9.2290	9.2645	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6590	9.6953
54	9.9144	9.9512	9.9880	10.0249	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7387	10.7769	10.8153
57	11.0466	11.0854	11.1243	11.1632	11.2022
58	11.4376	11.4771	11.5166	11.5562	11.5959
59	11.8354	11.8756	11.9158	11.9561	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037

3.5	4.6	5.7	6.8	7.9
3.3736	3.3951	3.4166	3.4382	3.4599
3.5912	3.6134	3.6356	3.6579	3.6802
3.8156	3.8385	3.8613	3.8843	3.9073
4.0468	4.0703	4.0939	4.1175	4.1412
4.2848	4.3090	4.3333	4.3576	4.3820
4.5296	4.5545	4.5794	4.6044	4.6295
4.7812	4.8068	4.8324	4.8581	4.8838
5.0396	5.0659	5.0921	5.1185	5.1449
5.3048	5.3317	5.3587	5.3857	5.4128
5.5768	5.6044	5.6321	5.6598	5.6876
5.8556	5.8839	5.9122	5.9406	5.9691
6.1412	6.1702	6.1992	6.2283	6.2574
6.4336	6.4633	6.4929	6.5227	6.5525
6.7328	6.7634	6.7935	6.8239	6.8544
7.0388	7.0698	7.1009	7.1320	7.1632
7.3516	7.3835	7.4155	7.4468	7.4787
7.6712	7.7036	7.7360	7.7685	7.8010
7.9976	8.0307	8.0637	8.0969	8.1301
8.3308	8.3645	8.3983	8.4321	8.4660
8.6708	8.7052	8.7397	8.7742	8.8088
9.0176	9.0527	9.0878	9.1230	9.1583
9.3712	9.4070	9.4428	9.4787	9.5146
9.7316	9.7681	9.8045	9.8411	9.8777
10.0988	10.1359	10.1731	10.2103	10.2476
10.4728	10.5106	10.5485	10.5864	10.6244
10.8536	10.8921	10.9306	10.9692	11.0079
11.2412	11.2804	11.3196	11.3589	11.3982
11.6356	11.6755	11.7153	11.7553	11.7953
12.0368	12.0773	12.1179	12.1585	12.1992
12.4448	12.4860	12.5273	12.5686	12.6100

TABLE

I

THE foregoing Table is useful in Gauging Wine Vessels.

Example. There is a *Brandy Butt* supposed to be in the Form of the middle Frustum of a Sphere. The Bung-Diameter is 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches. What is the Content of this Cask in Wine-Gallons?

From the Bung-Diameter $\xrightarrow{\hspace{1.5cm}}$ 32
Subtract the Head-Diameter $\xrightarrow{\hspace{1.5cm}}$ 24

The Difference is $\xrightarrow{\hspace{1.5cm}}$ 8

Find 8 in the first Column of the Table, pag. 1 against it is 5.6, which added to 24, the Head-Diameter is 29.6 the Mean Diameter of this Cask. Now against 29 in this Table, and under 40 Length of the Cask, gives 119.156, the Content sought. The like for any other Cask whose Mean Diameter is known.

291.2	281.2	270.2	250.2	230.2
241.2	231.2	220.2	200.2	180.2
191.2	181.2	170.2	150.2	130.2
141.2	131.2	120.2	100.2	80.2
31.2	21.2	10.2	0.2	
291.2	281.2	270.2	250.2	230.2
241.2	231.2	220.2	200.2	180.2
191.2	181.2	170.2	150.2	130.2
141.2	131.2	120.2	100.2	80.2
31.2	21.2	10.2	0.2	
291.2	281.2	270.2	250.2	230.2
241.2	231.2	220.2	200.2	180.2
191.2	181.2	170.2	150.2	130.2
141.2	131.2	120.2	100.2	80.2
31.2	21.2	10.2	0.2	

TABLE

TABLES
OF
EXCISE,

According to the
Respective Allowances

Made to
Common Brewers,

Both in
LONDON

And the
COUNTRY.

Excise for Common Brewers in the County, at 4 s. 9 d. per Barrel, the Allowances being $2\frac{1}{2}$ Barrels in every 23.

0					$\frac{1}{2}$					$\frac{1}{2}$					$\frac{1}{4}$									
B.	l.	s.	d.	f.	23	B.	l.	s.	d.	f.	23	B.	l.	s.	d.	f.	23	B.	l.	s.	d.	f.	23	
10	4	23	50			5	32	06	6	40	19	0	7	43				0	7	43				
20	8	52	100			9	61	50	10	70	10	0	11	72				0	11	72				
30	12	81	150			13	90	100	14	93	60	0	15	102				0	15	102				
40	16	110	200			17	113	150	19	02	11	1	0	11				1	0	11				
50	20	139	250			21	142	200	23	31	16	1	4	40				1	4	40				
60	24	168	300			25	171	250	27	60	21	1	8	63				1	8	63				
70	28	197	350			29	200	300	31	89	30	3	12	92				3	12	92				
80	32	226	400			33	229	350	37	18	39	3	17	01				3	17	01				
90	36	255	450			37	258	400	41	47	48	3	22	31				3	22	31				
100	40	284	500			41	287	450	45	76	57	3	27	60				3	27	60				
110	44	313	550			45	316	500	49	105	06	3	32	89				3	32	89				
120	48	342	600			49	345	550	53	134	15	3	37	18				3	37	18				
130	52	371	650			53	374	600	57	163	24	3	42	47				3	42	47				
140	56	400	700			57	403	650	61	192	33	3	47	76				3	47	76				
150	60	429	750			61	432	700	65	221	42	3	52	105				3	52	105				
160	64	458	800			65	461	750	69	250	51	3	57	134				3	57	134				
170	68	487	850			69	490	800	73	279	60	3	62	163				3	62	163				
180	72	516	900			73	519	850	77	308	69	3	67	192				3	67	192				
190	76	545	950			77	548	900	81	337	78	3	72	221				3	72	221				
200	80	574	1000			81	577	950	85	366	87	3	77	250				3	77	250				
210	84	603	1050			85	606	1000	89	395	96	3	82	279				3	82	279				
220	88	632	1100			89	635	1050	93	424	105	3	87	308				3	87	308				
230	92	661	1150			93	664	1100	97	453	114	3	92	337				3	92	337				
240	96	690	1200			97	693	1150	101	482	123	3	97	366				3	97	366				
250	100	719	1250			101	722	1200	105	511	132	3	102	395				3	102	395				
260	104	748	1300			105	751	1250	109	540	141	3	107	424				3	107	424				
270	108	777	1350			109	780	1300	113	569	150	3	112	453				3	112	453				
280	112	806	1400			113	809	1350	117	598	159	3	117	482				3	117	482				
290	116	835	1450			117	838	1400	121	627	168	3	122	511				3	122	511				
300	120	864	1500			121	867	1450	125	656	177	3	127	540				3	127	540				
310	124	893	1550			125	896	1500	129	685	186	3	132	569				3	132	569				
320	128	922	1600			129	925	1550	133	714	195	3	137	598				3	137	598				
330	132	951	1650			133	954	1600	137	743	204	3	142	627				3	142	627				
340	136	980	1700			137	983	1650	141	772	213	3	147	656				3	147	656				
350	140	1009	1750			141	1012	1700	145	801	222	3	152	685				3	152	685				
360	144	1038	1800			145	1041	1750	149	830	231	3	157	714				3	157	714				
370	148	1067	1850			149	1070	1800	153	859	240	3	162	743				3	162	743				
380	152	1096	1900			153	1099	1850	157	888	249	3	167	772				3	167	772				
390	156	1125	1950			157	1128	1900	161	917	258	3	172	801				3	172	801				
400	160	1154	2000			161	1157	1950	165	946	267	3	177	830				3	177	830				
410	164	1183	2050			165	1186	2000	169	975	276	3	182	859				3	182	859				
420	168	1212	2100			169	1215	2050	173	1004	285	3	187	888				3	187	888				
430	172	1241	2150			173	1244	2100	177	1033	294	3	192	917				3	192	917				
440	176	1270	2200			177	1273	2150	181	1062	303	3	197	946				3	197	946				
450	180	1299	2250			181	1302	2200	185	1091	312	3	202	975				3	202	975				
460	184	1328	2300			185	1331	2250	189	1120	321	3	207	1004				3	207	1004				
470	188	1357	2350			189	1360	2300	193	1149	330	3	212	1033				3	212	1033				
480	192	1386	2400			193	1389	2350	197	1178	339	3	217	1062				3	217	1062				
490	196	1415	2450			197	1418	2400	201	1207	348	3	222	1091				3	222	1091				
500	200	1444	2500			201	1447	2450	205	1236	357	3	227	1120				3	227	1120				

COUNTY

Exc

Excise for Common Brewers in the Country,
at 4 s. 9 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
23	4	17	4	2	713	150	18	7	2
46	9	14	9	0	736	155	16	0	0
69	14	12	1	2	759	160	13	4	2
92	19	09	6	0	782	165	10	9	0
115	24	6	10	2	805	170	8	1	2
138	29	4	3	0	828	175	5	6	0
161	34	1	7	2	851	180	2	10	2
184	38	19	0	0	874	185	0	3	0
207	43	16	4	2	897	189	17	7	2
230	48	13	9	0	920	194	15	0	0
253	53	11	1	2	943	199	12	4	2
276	58	8	6	0	966	204	9	9	0
299	63	5	10	2	989	209	7	1	2
322	68	3	3	0	1012	214	4	6	0
345	73	0	7	2	1035	219	1	10	2
368	77	18	0	0	1058	223	19	3	0
391	82	15	4	2	1081	228	16	7	2
414	87	12	9	0	1104	223	14	0	0
437	92	10	1	2	1127	238	11	4	2
460	97	7	6	0	1150	242	8	9	0
483	102	4	10	2	1173	248	6	1	2
506	107	2	3	0	1196	253	3	6	0
529	111	19	7	2	1219	258	0	10	2
552	116	17	0	0	1242	262	18	3	0
575	121	14	4	2	1265	267	15	7	2
598	126	11	9	0	1288	272	13	0	0
621	131	9	1	2	1311	277	10	4	2
644	136	6	6	0	1334	282	7	9	0
667	141	3	10	2	1357	287	5	1	2
690	146	1	3	0	1380	292	2	6	0

Excise for Common Brewers in the County,
at 4 s. 9 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	296	19	10	2	2093	443	1	1	2
1426	301	17	3	0	2116	447	18	6	0
1449	306	14	7	2	2139	452	15	10	2
1472	311	12	0	0	2162	457	13	3	0
1495	316	9	4	2	2185	462	10	7	2
1518	321	6	9	0	2208	467	8	0	0
1541	326	4	1	2	2231	472	5	4	2
1564	331	1	6	0	2254	477	2	9	0
1587	335	18	10	2	2277	482	0	1	2
1610	340	16	3	0	2300	486	17	6	0
1633	345	13	7	2	2323	491	14	10	2
1656	350	11	0	0	2346	496	12	3	0
1679	355	8	4	2	2369	501	9	7	2
1702	360	5	9	0	2392	506	7	0	0
1725	365	3	1	2	2415	511	4	4	2
1748	370	0	6	0	2438	516	1	9	0
1771	374	17	10	2	2461	520	19	1	2
1794	379	15	3	0	2484	525	16	6	0
1817	384	12	7	2	2507	530	13	10	2
1840	389	10	0	0	2530	535	11	3	0
1863	394	7	4	2	5060	1071	2	6	0
1886	399	4	9	0	7590	1636	13	9	0
1909	404	2	1	2	10120	2142	5	0	0
1932	408	19	6	0	12650	2677	16	3	0
1955	413	16	10	2	15180	3213	7	6	0
1978	418	14	3	0	17710	3748	18	9	0
2001	423	11	7	2	20240	4284	10	0	0
2024	428	9	0	0	22770	4820	1	3	0
2047	433	6	4	2	25300	5355	12	6	0
2070	438	3	9	0	27830	5891	3	9	0

Excise

Excise for Common Brewers in the Country, at 1 s. 3 d. per Barrel, the Allowances being $2\frac{1}{2}$ Barrels in every 13.

0				1				2				3				4				5				6				7				8				9			
s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23				s. d. f. 23			
0	1	1	11	0	2	6	0	0	1	8	0	0	1	8	0	0	1	8	0	0	1	8	0	0	1	8	0	0	1	8	0	0	1	8	0	0			
0	2	2	22	0	3	6	0	0	2	8	0	0	2	8	0	0	2	8	0	0	2	8	0	0	2	8	0	0	2	8	0	0	2	8	0	0			
0	3	4	0	0	3	7	1	0	0	3	10	3	4	0	0	3	10	3	4	0	0	3	10	3	4	0	0	3	10	3	4	0	0	3	10	3	4		
0	4	5	1	0	4	8	3	0	0	4	11	6	0	0	0	4	11	6	0	0	0	4	11	6	0	0	0	4	11	6	0	0	0	4	11	6	0		
0	5	6	3	0	5	10	0	17	0	5	12	3	0	0	0	5	12	3	0	0	0	5	12	3	0	0	0	5	12	3	0	0	0	5	12	3	0		
0	6	8	0	0	6	11	2	5	0	6	13	2	0	0	0	6	13	2	0	0	0	6	13	2	0	0	0	6	13	2	0	0	0	6	13	2	0		
0	7	9	2	0	7	12	0	16	0	7	14	0	0	0	0	7	14	0	0	0	0	7	14	0	0	0	0	7	14	0	0	0	0	7	14	0	0		
0	8	10	3	0	8	13	1	4	0	8	15	1	0	0	0	8	15	1	0	0	0	8	15	1	0	0	0	8	15	1	0	0	0	8	15	1	0		
0	9	11	4	0	9	14	2	15	0	9	16	2	0	0	0	9	16	2	0	0	0	9	16	2	0	0	0	9	16	2	0	0	0	9	16	2	0		
0	10	12	5	0	10	15	3	0	3	10	17	3	0	0	0	10	17	3	0	0	0	10	17	3	0	0	0	10	17	3	0	0	0	10	17	3	0		
0	11	13	6	0	11	16	4	0	3	11	18	4	0	0	0	11	18	4	0	0	0	11	18	4	0	0	0	11	18	4	0	0	0	11	18	4	0		
0	12	14	7	0	12	17	5	14	0	12	19	5	0	0	0	12	19	5	0	0	0	12	19	5	0	0	0	12	19	5	0	0	0	12	19	5	0		
0	13	15	8	0	13	18	6	13	0	13	20	6	0	0	0	13	20	6	0	0	0	13	20	6	0	0	0	13	20	6	0	0	0	13	20	6	0		
0	14	16	9	0	14	19	7	13	0	14	21	7	0	0	0	14	21	7	0	0	0	14	21	7	0	0	0	14	21	7	0	0	0	14	21	7	0		
0	15	17	10	0	15	20	8	12	0	15	22	8	0	0	0	15	22	8	0	0	0	15	22	8	0	0	0	15	22	8	0	0	0	15	22	8	0		
0	16	18	11	0	16	21	9	12	0	16	23	9	0	0	0	16	23	9	0	0	0	16	23	9	0	0	0	16	23	9	0	0	0	16	23	9	0		
0	17	19	12	0	17	22	10	0	0	17	24	10	0	0	0	17	24	10	0	0	0	17	24	10	0	0	0	17	24	10	0	0	0	17	24	10	0		
0	18	20	13	0	18	23	11	0	0	18	25	11	0	0	0	18	25	11	0	0	0	18	25	11	0	0	0	18	25	11	0	0	0	18	25	11	0		
0	19	21	14	0	19	24	12	11	0	19	26	12	0	0	0	19	26	12	0	0	0	19	26	12	0	0	0	19	26	12	0	0	0	19	26	12	0		
0	20	22	15	0	20	25	13	12	0	20	27	13	0	0	0	20	27	13	0	0	0	20	27	13	0	0	0	20	27	13	0	0	0	20	27	13	0		
0	21	23	16	0	21	26	14	10	0	21	28	14	0	0	0	21	28	14	0	0	0	21	28	14	0	0	0	21	28	14	0	0	0	21	28	14	0		
0	22	24	17	0	22	27	15	10	0	22	29	15	0	0	0	22	29	15	0	0	0	22	29	15	0	0	0	22	29	15	0	0	0	22	29	15	0		
0	23	25	18	0	23	28	16	9	0	23	30	16	0	0	0	23	30	16	0	0	0	23	30	16	0	0	0	23	30	16	0	0	0	23	30	16	0		
0	24	26	19	0	24	29	17	8	0	24	31	17	0	0	0	24	31	17	0	0	0	24	31	17	0	0	0	24	31	17	0	0	0	24	31	17	0		
0	25	27	20	0	25	30	18	7	0	25	32	18	0	0	0	25	32	18	0	0	0	25	32	18	0	0	0	25	32	18	0	0	0	25	32	18	0		
0	26	28	21	0	26	31	19	6	0	26	33	19	0	0	0	26	33	19	0	0	0	26	33	19	0	0	0	26	33	19	0	0	0	26	33	19	0		
0	27	29	22	0	27	32	20	5	0	27	34	20	0	0	0	27	34	20	0	0	0	27	34	20	0	0	0	27	34	20	0	0	0	27	34	20	0		
0	28	30	23	0	28	33	21	4	0	28	35	21	0	0	0	28	35	21	0	0	0	28	35	21	0	0	0	28	35	21	0	0	0	28	35	21	0		
0	29	31	24	0	29	34	22	3	0	29	36	22	0	0	0	29	36	22	0	0	0	29	36	22	0	0	0	29	36	22	0	0	0	29	36	22	0		
0	30	32	25	0	30	35	23	2	0	30	37	23	0	0	0	30	37	23	0	0	0	30	37	23	0	0	0	30	37	23	0	0	0	30	37	23	0		
0	31	33	26	0	31	36	24	1	0	31	38	24	0	0	0	31	38	24	0	0	0	31	38	24	0	0	0	31	38	24	0	0	0	31	38	24	0		
0	32	34	27	0	32	37	25	0	0	32	39	25	0	0	0	32	39	25	0	0	0	32	39	25	0	0	0	32	39	25	0	0	0	32	39	25	0		
0	33	35	28	0	33	38	26	0	0	33	40	26	0	0	0	33	40	26	0	0	0	33	40	26	0	0	0	33	40	26	0	0	0	33	40	26	0		
0	34	36	29	0	34	39	27	0	0	34	41	27	0	0	0	34	41	27	0	0	0	34	41	27	0	0	0	34	41	27	0	0	0	34	41	27	0		
0	35	37	30	0	35	40	28	0	0	35	42	28	0	0	0	35	42	28	0	0	0	35	42	28	0	0	0	35	42	28	0	0	0	35	42	28	0		
0	36	38	31	0	36	41	29	0	0	36	43	29	0	0	0	36	43	29	0	0	0	36	43	29	0	0	0	36	43	29	0	0	0	36	43	29	0		
0	37	39	32	0	37	42	30	0	0	37	44	30	0	0	0	37	44	30	0	0	0	37	44	30	0	0	0	37	44	30	0	0	0	37	44	30	0		
0	38	40	33	0	38	43	31	0	0	38	45	31	0	0	0	38	45	31	0	0	0	38	45	31	0	0	0	38	45	31	0	0	0	38	45	31	0		
0	39	41	34	0	39	44	32	0	0	39	46	32	0	0	0	39	46	32	0	0	0	39	46	32	0	0	0	39	46	32	0	0	0	39	46	32	0		
0	40	42	35	0	40	45	33	0	0	40	47	33	0	0	0	40	47	33	0	0	0	40	47	33	0	0	0	40	47	33	0	0	0	40	47	33	0		
0	41	43	36	0	41	46	34	0	0	41	48	34	0	0	0	41	48	34	0	0	0	41	48	34	0	0	0	41	48	34	0	0	0	41	48	34	0		
0	42	44	37	0	42	47	35	0	0	42	49	35	0	0	0	42	49	35	0	0	0	42	49	35	0	0	0	42	49	35	0	0	0	42	49	35	0		
0	43	45	38	0	43	48	36	0	0	43	50	36	0	0	0	43	50	36	0	0	0	43	50	36	0	0	0	43	50	36	0	0	0	43	50	36	0		
0	44	46	39	0	44	49	37	0	0	44	51	37	0	0	0	44	51	37	0	0	0	44	51	37	0	0	0	44	51	37	0	0	0	44	51	37	0		
0	45	47	40																																				

Excise

Excise for Common Brewers in the Country,
at 1 s. 3 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
23	1	5	7	2	713	39	14	4	2
46	2	11	3	0	736	41	0	0	0
69	3	16	10	2	759	42	5	7	2
92	5	2	6	0	782	43	11	3	0
115	6	8	1	2	805	44	16	10	2
138	7	13	9	0	828	46	2	6	0
161	8	19	4	2	851	47	8	1	2
184	10	5	0	0	874	48	13	9	0
207	11	10	7	2	897	49	19	4	2
230	12	16	3	0	920	51	5	0	0
253	14	1	10	2	943	52	10	7	2
276	15	7	6	0	966	53	16	3	0
299	16	13	1	2	989	55	1	10	2
322	17	18	9	0	1012	56	7	6	0
345	19	4	4	2	1035	57	13	1	2
368	20	10	0	0	1058	58	18	9	0
391	21	15	7	2	1081	60	4	4	2
414	23	1	3	0	1104	61	10	0	0
437	24	6	10	2	1127	62	15	7	2
460	25	12	6	0	1150	64	1	3	0
483	26	18	1	2	1173	65	6	10	2
506	28	3	9	0	1196	66	12	6	0
529	29	9	4	2	1219	67	18	1	2
552	30	15	0	0	1242	69	3	9	0
575	32	0	7	2	1265	70	9	4	2
598	33	6	3	0	1288	71	15	0	0
621	34	11	10	2	1311	73	0	7	2
644	35	17	6	0	1334	74	6	3	0
667	37	3	1	2	1357	75	11	10	2
690	38	8	9	0	1380	76	17	6	0

Excise

Excise for Common Brewers in the Country,
at 1 s. 3 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	78	3	1	2	2093	116	11	10	0
1426	79	8	9	0	2116	117	17	6	0
1449	80	14	4	2	2139	119	3	1	2
1472	82	0	0	0	2162	120	8	9	0
1495	83	5	7	2	2185	121	14	4	2
1518	84	11	3	0	2208	123	0	0	0
1541	85	16	10	2	2231	124	5	7	2
1564	87	2	6	0	2254	125	11	3	0
1587	88	8	1	2	2277	126	16	10	2
1610	89	13	9	0	2300	128	2	6	0
1633	90	19	4	2	2323	129	8	1	2
1656	92	5	0	0	2346	130	13	9	0
1679	93	10	7	2	2369	131	19	4	2
1702	94	16	3	0	2392	133	5	0	0
1725	96	1	10	2	2415	134	10	7	2
1748	97	7	6	0	2438	135	16	3	0
1771	98	13	1	2	2461	137	1	10	2
1794	99	18	9	0	2484	138	7	6	0
1817	101	4	4	2	2507	139	13	1	2
1840	102	10	0	0	2530	140	18	9	0
1863	103	15	7	2	5060	281	17	6	0
1886	105	1	3	0	7590	422	16	3	0
1909	106	6	10	2	10120	563	15	0	0
1932	107	12	6	0	12650	704	13	9	0
1955	108	18	1	2	15180	845	12	6	0
1978	110	3	9	0	17710	986	11	3	0
2001	111	9	4	2	20240	1127	10	0	0
2024	112	15	0	0	22770	1268	8	9	0
2047	114	0	7	2	25300	1409	7	6	0
2070	115	6	3	0	27830	1550	6	3	0

Excise for Common Brewers in London
at 4 s. 9 d. per Barrel, the Allowances
being 3 Barrels in every 23 of Beer.

B.	s.	d.	f.	23	s.	d.	f.	23	s.	d.	f.	23	s.	d.	f.	23
1	4	12	60	5	13	19	0	6	21	90	7	22	22	0	0	0
2	8	30	120	9	32	20	10	33	15	0	11	41	0	0	0	0
3	12	42	180	13	50	80	14	51	21	0	15	53	11	0	0	0
4	16	61	10	17	62	140	18	70	40	0	19	71	17	0	0	0
5	0	73	71	1	80	20	1	82	10	1	3	90	0	0	0	0
6	4	91	131	5	93	31	6	100	16	1	7	102	0	0	0	0
7	8	103	191	9	111	91	10	112	22	1	12	00	19	0	0	0
8	13	02	21	14	03	19	15	11	31	1	16	12	1	0	0	0
9	17	20	81	18	21	21	19	23	12	2	0	31	1	0	0	0
10	1	32	142	2	40	40	3	41	17	2	4	43	7	0	0	0
11	5	50	202	6	52	102	7	60	02	8	61	11	0	0	0	0
12	9	63	32	10	70	162	11	72	62	12	73	19	0	0	0	0
13	13	81	92	14	82	222	15	90	122	16	92	2	0	0	0	0
14	17	93	192	18	101	32	19	102	183	0	110	8	0	0	0	0
15	1	111	213	2	113	113	4	01	13	5	02	16	0	0	0	0
16	6	10	43	7	11	173	8	13	73	9	20	20	0	0	0	0
17	10	22	103	11	30	03	12	31	133	13	33	9	0	0	0	0
18	14	40	163	15	42	63	16	43	193	17	51	9	0	0	0	0
19	18	52	223	19	60	124	0	62	24	1	63	11	0	0	0	0
20	2	71	54	3	72	184	4	80	04	5	81	21	0	0	0	0
21	6	83	114	7	11	14	8	92	44	9	100	4	0	0	0	0
22	10	101	174	11	103	74	12	110	204	13	112	10	0	0	0	0

Excise

Excise for Common Brewers in London, at
4 s. 9 d. per Barrel, &c. Beer. 18

Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.
23	4	15	713	147	5	1403	289	15	2093	432	5
46	9	10	736	152	0	1426	294	10	2116	437	0
69	14	5	759	156	15	1449	299	5	2139	441	15
92	19	0	782	161	10	1472	304	0	2162	446	10
115	23	15	804	166	5	1495	308	15	2185	451	5
138	28	10	828	171	0	1518	313	10	2208	456	0
161	33	5	851	175	15	1541	318	5	2231	460	15
184	38	0	874	180	10	1564	323	0	2254	465	10
207	42	15	897	185	5	1587	327	15	2277	470	5
230	47	10	920	190	0	1610	332	10	2300	475	10
253	52	5	943	194	15	1633	337	5	2323	479	15
276	57	0	966	199	10	1656	342	0	2346	484	10
299	61	15	989	204	5	1679	346	15	2369	489	5
322	66	10	1012	209	0	1702	351	10	2392	494	0
345	71	5	1035	213	15	1725	356	5	2415	498	15
368	76	0	1058	218	10	1748	361	0	2438	503	10
391	80	15	1081	223	5	1771	365	15	2461	508	5
414	85	10	1104	228	0	1794	370	10	2484	513	0
437	90	5	1127	232	15	1817	375	5	2507	517	15
460	95	0	1150	237	10	1840	380	0	2530	522	10
483	99	15	1173	242	5	1863	384	15	2553	527	5
506	104	10	1196	247	0	1886	389	10	2576	532	10
529	109	5	1219	251	15	1909	394	5	2599	537	5
552	114	0	1242	256	10	1932	399	0	2622	542	0
575	118	15	1265	261	5	1955	403	15	2645	547	5
598	123	10	1288	266	0	1978	408	10	2668	552	10
621	128	5	1311	270	15	2001	413	5	2691	557	5
644	133	0	1334	275	10	2024	418	0	2714	562	0
667	137	15	1357	280	5	2047	422	15	2737	567	5
690	142	10	1380	285	0	2070	427	10	2760	572	10

Excise

Excise for Common Brewers in London,
at 4 s. 9 d. per Barrel, the Allowances
being 2 Barrels in every 22 of Ale.

B.	0				$\frac{1}{4}$				$\frac{1}{2}$				$\frac{3}{4}$			
	s.	d.	f.	22l.	s.	d.	f.	22l.	s.	d.	f.	22l.	s.	d.	f.	22l.
1	3	4	33	60	5	43	20	6	52	20	0	7	62	16		
2	8	72	12	0	9	82	8	10	92	4	0	11	102	0		
3	12	11	1	18	14	01	14	15	11	10	0	16	21	6		
4	17	31	20	0	18	40	20	19	50	16	1	0	60	12		
5	1	70	81	1	2	80	40	3	90	0	1	4	93	18		
6	5	103	14	1	6	113	10	8	03	61	9	13	2			
7	10	22	20	1	11	32	16	12	42	12	1	13	52	8		
8	14	62	4	1	15	72	0	16	81	18	1	17	91	14		
9	18	101	10	1	19	111	62	1	01	22	2	2	1020			
10	23	20	16	2	4	30	12	5	40	82	6	50	4			
11	7	60	02	2	8	63	18	9	73	14	2	10	83	18		
12	11	93	62	2	12	103	28	13	112	20	2	15	03	16		
13	16	12	12	2	17	22	82	18	32	42	19	42	0			
14	0	51	18	3	1	61	14	2	71	10	3	3	81	6		
15	4	91	23	3	5	100	20	3	6	110	16	8	00	12		
16	9	10	8	3	10	20	43	11	30	03	12	33	18			
17	13	43	14	3	14	53	10	15	63	63	16	73	2			
18	17	82	20	3	18	92	16	19	102	124	0	112	8			
19	2	02	44	4	3	12	04	4	21	184	5	31	14			
20	6	41	10	4	7	51	64	8	61	24	9	70	20			
21	10	80	16	4	11	90	124	12	100	84	13	110	4			

Excise

Excise for Common Brewers in London, at
4 s. 9 d. per Barrel, &c. Ale.

Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.
22	4	15	682	147	5	1342	289	15	2002	432	5
44	9	10	704	152	0	1364	294	10	2024	437	0
66	14	5	726	156	15	1386	299	5	2046	441	15
88	19	0	748	161	10	1408	304	0	2068	445	10
110	23	15	770	166	5	1430	308	15	2090	451	5
132	28	10	792	171	0	1452	313	10	2112	456	0
154	33	5	814	175	15	1474	318	5	2134	460	15
176	38	0	835	180	10	1496	323	0	2156	465	10
198	42	15	858	185	5	1518	327	15	2178	470	5
220	47	10	880	190	0	1540	332	10	2200	475	0
242	52	5	902	194	15	1562	337	5	2222	479	15
264	57	0	924	199	10	1584	342	0	2244	484	10
286	61	15	946	204	5	1606	346	15	2266	489	5
308	66	10	968	209	0	1628	351	10	2288	494	0
330	71	5	990	213	15	1650	356	5	2310	498	15
352	76	0	1012	218	10	1672	361	0	2332	503	10
374	80	15	1034	223	5	1694	365	15	2354	508	5
396	85	10	1056	228	0	1716	370	10	2376	513	0
418	90	5	1078	232	15	1738	375	5	2398	517	15
440	95	0	1100	237	10	1760	380	0	2420	522	10
462	99	15	1122	242	5	1782	384	15	4840	1045	0
484	104	10	1144	247	0	1804	389	10	7260	1567	10
506	109	5	1166	251	15	1826	394	5	9680	2090	0
528	114	0	1188	256	10	1848	399	0	12100	2612	10
550	118	15	1210	261	5	1870	403	15	14520	3135	0
572	123	10	1232	266	0	1892	408	10	16940	3657	10
594	128	5	1254	270	15	1914	413	5	19360	4180	0
616	133	0	1276	275	10	1936	418	0	21780	4702	10
638	137	15	1298	280	5	1958	422	15	24200	5225	0
660	142	10	1320	285	0	1980	427	10	26620	5747	10

Excise

Excise for Common Brewers in London,
at 1 s. 3 d. per Barrel, the Allowances
being 3 Barrels in every 23 *Small-Beer*.

0				$\frac{1}{4}$				$\frac{1}{2}$				$\frac{3}{4}$			
B.	l.	s.	d.	f.	23	l.	s.	d.	f.	23	l.	s.	d.	f.	23
10	1	10	40			1	41	50			1	72	60		1 103 7
20	2	20	80			2	51	90			2	82	100		2 113 11
30	3	30	120			3	61	130			3	92	140		3 63 15
40	4	40	160			4	71	170			4	102	180		4 13 19
50	5	50	200			5	81	210			5	112	220		5 30 0
60	6	61	10			6	92	20			6	03	30		6 40 4
70	7	71	50			7	102	60			7	13	70		7 50 8
80	8	81	90			8	112	100			8	23	110		8 60 12
90	9	91	130			9	122	140			9	33	150		9 70 16
100	10	101	170			10	132	180			10	43	190		10 80 20
110	11	111	210			11	142	220			11	53	230		11 91 1
120	12	121	250			12	152	260			12	60	00		12 91 1
130	13	131	290			13	162	300			13	70	40		13 101 5
140	14	141	330			14	172	340			14	80	80		14 111 9
150	15	151	370			15	182	380			15	90	120		15 121 13
160	16	161	410			16	192	420			16	100	160		16 131 17
170	17	171	450			17	202	460			17	110	200		17 141 21
180	18	181	490			18	212	500			18	120	240		18 151 25
190	19	191	530			19	222	540			19	130	280		19 161 29
200	20	201	570			20	232	580			20	140	320		20 171 33
210	21	211	610			21	242	620			21	150	360		21 181 37
220	22	221	650			22	252	660			22	160	400		22 191 41

Excise

To value of old to new

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Excise for Common Brewers in London, at
1 s. 3 d. per Bar. &c. Small-Beer.

Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.	Bar.	l.	s.
23	1	5	712	38	15	1403	76	5	2092	113	15
46	2	10	726	40	0	1426	77	10	2116	115	0
69	3	15	759	41	5	1449	78	15	2139	116	5
92	5	0	782	42	10	1472	80	0	2162	117	10
115	6	5	805	43	15	1495	81	5	2185	118	15
138	7	10	828	45	0	1518	82	10	2208	120	0
161	8	15	851	46	5	1541	83	15	2231	121	5
184	10	0	874	47	10	1564	85	0	2254	122	10
207	11	5	897	48	15	1587	86	5	2277	123	15
230	12	10	920	50	0	1610	87	10	2300	125	0
253	13	15	943	51	5	1633	88	15	2323	126	5
276	15	0	966	52	10	1656	90	0	2346	127	10
299	16	5	989	53	15	1679	91	5	2369	128	15
322	17	10	1012	55	0	1702	92	10	2392	130	0
345	18	15	1035	56	5	1725	93	15	2415	131	5
368	20	0	1058	57	10	1748	95	0	2438	132	10
391	21	5	1081	58	15	1771	96	5	2461	133	15
414	22	10	1104	60	0	1794	97	10	2484	135	0
437	23	15	1127	61	5	1817	98	15	2507	136	5
460	25	0	1150	62	10	1840	100	0	2530	137	10
483	26	5	1173	63	15	1863	101	5	5060	275	0
506	27	10	1196	65	0	1886	102	10	7590	412	10
529	28	15	1219	66	5	1909	103	15	10120	550	0
552	30	0	1242	67	10	1932	105	0	12650	687	10
575	31	5	1265	68	15	1955	106	5	15180	825	0
598	32	10	1288	70	0	1978	107	10	17710	962	10
621	33	15	1311	71	5	2001	108	15	20240	1100	0
644	35	0	1334	72	10	2024	110	0	22770	1237	10
667	36	5	1357	73	15	2047	111	5	25300	1375	0
690	37	10	1380	75	0	2070	112	10	27830	1512	10

The

The foregoing Tables (though very short) will shew the exact Duty for any Number of Barrels; and their use will be render'd very plain and easie, by the Directions and Example following.

1. If the number of Barrels of strong or small Beer given be less than 23, seek it in the first Column of the Table, and against it you have the exact Duty in Pounds, Shillings, Pence, Farthings, and 23th parts of a Farthing. Thus by the first Table the Duty of 10 Barrels and a half will be found to be 2 *l.* 4 *s.* 5 *d.* 1 *f.* and $\frac{1}{23}$ parts of a Farthing; for against 10 in the Column of Barrels, and under $\frac{1}{2}$ you have this Sum.

2. For any Number above 23; seek it in some of the Columns of Barrels in the Table, and if you cannot find the same Number, take the next less, and set down the Sum of Money against it, then subtract the Number found from the Number given, the Remainder (which will ever be less than 23) being found in the first Column of the Table, gives you a Sum of Money, which added to the former is the exact Duty required.

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Example.

Example.

What Sum of Money will the Duty of 1306½
Barrels of Strong Beer amount unto?

See the Work by the first Table.

Number given — 1306½ l. s. d. f.
Next less per Table — 1288 = 276 : 13 : 00 : 0 : 00
Remainder — 18½ = 3 : 18 : 3 : 3 : 12

The exact Duty is — 276 : 11 : 3 : 3 : 12
That is 276 l. 11 s. 3 d. 3 f. and 11 parts of a Farthing.
And so for any other Number of Barrels by this
or any of the other Tables.

SECT.

S E C T. XI.

Short RULES

F O R —

Gauging of Malt.

THE Duty upon Malt is charged upon the Bushel, and so proportionably for a greater or lesser Quantity. The Bushel here intended is the *Winchester* Bushel, which (as settled by the first Act for laying a Duty upon Malt) contains 2150.42, solid Inches, but in Practice 2150, will be sufficiently near the Truth.

The Area of any plain Figure may be found in Inches, by the Rules laid down in *Section III*. Now if the Area in Inches be divided by 2150, the Quotient shews the Bushels and Parts which such Figure or Vessel contains at one Inch deep.

Example. There is a *Cistern* or *Uting* Fat, whose Base is a *Rectangular Parallelogram*, the Length is 72 Inches, and the Breadth 48 Inches, what is the Area or Content at one Inch deep? *Answer*, 1.607 ^{thou} is, one Bushel, and .607 thousand Parts of a Bushel. For 72 multiplied by 48, is 3456; and this divided by 2150, gives 1.607.

But

But the Area of any such Figure may be more readily found by the Lines A and B upon the Rule. Thus,

As 2150 upon A, is to one of the sides upon B;
So is the other side upon A, to the Area upon B.

So in the Example above :

Set 2150 upon A, to 72 upon B;

Then against 48 upon A, is 1.607 upon B :

Or set 2150 upon A, to 48 upon B;

Then against 72 upon A, is 1.607 upon B.

The Area thus found, being multiplied by the Mean Depth in Inches, gives the Content of such Cistern at that Depth ; and so for any quantity of Malt upon the Floor. But care must be taken in finding the Mean Depth of any quantity of Malt, either in the Cistern or upon the Floor, by reason of the Unevenness of the Surface of the Malt, and of the bottom of the Cistern or Floor upon which it is laid ; in order to which, take the Depth in 6, 8, or 10 Places ; add all these Depths together, and divide the Sum by the number of Places in which the Depths were taken, the Quotient will be the Mean Depth required.

Example. Suppose a Quantity of Malt upon a Floor, of which you take the Depth in Eight several Places, and find,

The

The	{	1	Depth to be	{	4.7	Inches.
		2			5.4	
		3			5.6	
		4			4.9	
		5			4.4	
		6			4.8	
		7			5.3	
		8			5.7	

The Sum of these is 40.8, which divided by 8, (the number of places) gives 5.1 for the Mean Depth.

Barley is sometimes steep'd in Round Vessels, and the Area of a Circle in Bushels is thus found. Take the Diameter in Inches, and let the Square thereof be multiplied by .0003643, or divided by 2737.47, the Product or Quotient will be the Area sought.

Example. Suppose the Diameter be 60 Inches, this squared is 3600, and this multiplied by .0003653, or divided by 2737.47, gives 1.315, the Area required.

But for any Round Vessel, the Rule gives the Area in Bushels by Inspection, by help of a certain Number call'd the *Gage-Point*, which in this case is 52.32, (being the Diameter of a Circle, whose Area is 2150, the number of solid Inches contain'd in the Bushel.) Thus in the Example above, the Diameter was 60 Inches; therefore, set the *Gage-Point*, (*viz.* 52.32) upon D, to 1 upon C, then against 60 upon D, is 1.315 the Area upon C. And the Rule being thus set, against any Diameter upon D, you have the Area upon C.

The

The Diameter and Depth of any Cylindrical Vessel being given in Inches, the Content in Bushels is readily found at once: Thus: Set the Gage-Point upon D, to the Depth upon C; then against the Diameter upon D, is the Content upon C.

In the last Example the Diameter was 60, suppose the Depth 40 Inches, the Content will be 52.6 Bushels, and so for any other.

Having above declared what the Content of the Winchester Bushel is, I shall here, for the satisfaction of the Inquisitive Reader, give an Account how the Content of the said Bushel came to be ascertained.

By the Statute of the 11 Hen. VII. Cap. 4. It is enacted, That Standard-Weights and Measures be made, and sent to the several Cities, Burroughs, and Market-Towns therein mentioned, which was accordingly done. But the Statute of the 12 Hen. VII. Cap. 5. recites, that upon more diligent Examination, the said Standard-Weights and Measures were found to be defective. And Enacts, That the Measure of a Bushel shall contain 8 Gallons of Wheat, and that every Gallon contain 8 Pounds Troy of Wheat, and that every Pound contain 12 Ounces Troy Weight, and every Ounce contain 20 *Sterlings* [now 20 Penny Weight] and every *Sterling* [or Penny Weight] be of the Weight of 32 Corns of Wheat, that grow in the middle of the Ear of Wheat. And that a Standard of a Bushel and a Gallon, after this Assize be made, and kept in the King's Treasury for ever. And new Bushels and Gallons were ordered to be made, and sent to the several Cities, Burroughs, &c. and the old ones brought and broken.

Accord-

Accordingly there remains now in the Custody the Chamberlaine of the Exchequer a Standard Brass Bushel, and a Standard Gallon agreeing thereon. Upon the Bushel there is this Inscription.

*Henricus Septimus Dei Gratia Rex
Angliae & Franciae.*

In February 1696. when a Bill was depending in Parliament, for laying a Duty upon Malt, I did with the assistance of Mr. George Toller, Mr. Philip Shales, and Mr. Tho. Jess, and in the Presence of several Members of the House of Commons, make an Experiment in order to find the true Content of the said Standard Bushel in the Exchequer. And the said Brass Bushel being exactly filled with common Spring Water, and the said Water being measured out again by a Regular Parallelepipedon (whose Base was four Inches square, and the Depth 14 Inches) was found to contain 2145.6 solid Inches. The said Water was also weighed by the Standard Weights in the Exchequer, (and by a Beam which would turn with six Grains, with thirty Pounds in each Scale) and found to be 1131 Ounces and a Penny Weight Troy.

Now to settle the Dimensions for a Bushel to answer the said Standard, these were agreed upon most convenient, viz. 18.5 Inches Diameter, and 10 Inches deep; for a Cylindrical Vessel of these Dimensions will contain 2150.42 solid Inches, which exceeding the Content of the Standard Bushel 4.82 Inches, and there being no other convenient Dimensions (without counting to the hundredth part of an Inch) that would come so near as these: It was Enacted, in the Act for laying Duty upon Malt,

That every Round Bushel, with a plain and even Bottom, being eighteen Inches and a half Diameter through-out, and eight Inches deep, shall be esteemed a legal Winchester Bushel, according to the Standard in His Majesty's Exchequer.

When this Experiment was made at the Exchequer, we also compared the Standard Troy Weights with the Standard Averdupoise Weights, and found that 14 Pound of Averdupoise, were equal to 18 Pound Ounces, and 15 Penny-weight Troy; so that 140 Ounces Averdupoise, are equal to 218.75 Ounces Troy, which in the least Terms in whole Numbers

Averdupoise	Troy
192	175 Ounces
144	125 Pounds

The Bushel, as now settled, contains 2150.42 Solid Inches, as was shewn above, and will contain of common Spring-Water 1134.344 Ounces Troy Per,

As 2145.6, the Inches in the Brass Standard, is to 1131.7 the Troy Ounces of Water it contained; So is 2150.42, the Content of a Bushel, as now settled, to 1134.344.

Hence the following Table shows the Content, in Solid Inches, of a Bushel, half Bushel, Peck, and Gallon, and likewise the weight of Water that each will contain.

Inches

	Troy.				Averdupoise.			
Inches.	lb.	ou.	p.	w.	lb.	ou.	dr.	
Bushel.	2150.42	94.	6.	6. 88	77.	12.	8.	60
Half Bush.	1075.21	47.	3.	3. 44	38.	14.	4.	30
Peck	537.60	23.	7.	11. 72	19.	7.	2.	15
Gallon	268.80	11.	9.	15. 86	9.	11.	9.	07

By this Table it is easie to make any of the Measures therein mentioned, or to try such as are already made, by the help of a good pair of Scales and true weight. But where these are not to be had the following Table shews the Diameters and Depth that will answer to the Bushel, half Bushel, Peck and Gallon, provided they be equally wide from the Top to the Bottom.

Example. To find whether a Bushel be equal to the Standard, or not, suppose the Diameter to

be 18 inches, the Inches in the Brass Standard is 18.7 the Troy Ounces of Water it contains; to 18.7 the Content of a Bushel, as now settled, is 1134.34.

Since the following Table shews the Content in old Inches of a Bushel, half Bushel, Peck, and Gallon, and likewise the weight of Water that each will contain.

Inches

Bushel.		Half Bu.		Peck.		Gallon.	
Diam.	Depth	Diam.	Depth	Diam.	Depth	Diam.	Depth
17.5	8.94	13.7	7.30	10.7	5.98	8.3	4.97
17.6	8.84	13.8	7.20	10.8	5.87	8.4	4.85
17.7	8.74	13.9	7.09	10.9	5.76	8.5	4.74
17.8	8.64	14.0	6.98	11.0	5.66	8.6	4.63
17.9	8.55	14.1	6.89	11.1	5.56	8.7	4.52
18.0	8.45	14.2	6.79	11.2	5.46	8.8	4.42
18.1	8.35	14.3	6.69	11.3	5.36	8.9	4.32
18.2	8.27	14.4	6.60	11.4	5.27	9.0	4.23
18.3	8.18	14.5	6.51	11.5	5.18	9.1	4.13
18.4	8.09	14.6	6.42	11.6	5.09	9.2	4.04
18.5	8.00	14.7	6.34	11.7	5.00	9.3	3.96
18.6	7.91	14.8	6.25	11.8	4.92	9.4	3.87
18.7	7.83	14.9	6.17	11.9	4.83	9.5	3.79
18.8	7.75	15.0	6.08	12.0	4.75	9.6	3.71
18.9	7.67	15.1	6.00	12.1	4.66	9.7	3.64
19.0	7.58	15.2	5.93	12.2	4.60	9.8	3.56
19.1	7.50	15.3	5.85	12.3	4.52	9.9	3.49
19.2	7.42	15.4	5.77	12.4	4.45	10.0	3.42
19.3	7.35	15.5	5.70	12.5	4.38	10.1	3.36
19.4	7.27	15.6	5.63	12.6	4.31	10.2	3.29
19.5	7.20	15.7	5.56	12.7	4.24	10.3	3.22

9 Inches, and one tenth part of an Inch, find this in the first Column, and against it you have 7.5, that is, seven Inches and a half; now if the Bushel be exactly so deep, it is a legal Bushel, if the Depth be more, it may be cut to this Depth. If the Bushel be not exactly round, and the Difference of the Diameters be not above an Inch,

K.

add

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add the longest and shortest Diameters together, and take half the Sum for the Mean Diameter, and proceed as above.

Suppose the Diameter of a Bushel be 19.5 Inches and the Depth but 7 Inches, you will find by the Table the Depth should be 7.2 Inches, from whence you may conclude this Bushel is less than the Standard; now to find how much it wants, say,

As 7.2 Inches, the Depth in the Table, is to 64 the Pints in a Bushel,

So is 7 Inches, the Depth of the Bushel given, to 62.22.

That is 62 Pints, and almost a quarter of a Pint, which subtracted from 64, the Remainder 1.77, that is, one Pint and three Quarters, and somewhat better, and so much this Bushel is too little.

Or thus, subtract 7, the depth of the Bushel given, from 7.2, the Depth found in the Table, the Difference is .2; then as $7.2:64::2:1.777$ as above.

The Use of the Tables for Half Bushels, Pecks and Gallons, are the same with respect to those Measures; and therefore Examples of these are needless.

SECT

S E C T. XII.

The USE of the
R U L E
In Questions of
Simple Interest.

Problem I.

find the Interest of any Sum of Money for any time proposed (in Years, Months, or Days) at any Rate of Interest.

1. For Years.

What is the Interest of 350 £ . for five Years, at 6 per Cent per Ann.

Set 1 upon A, to .06 the Rate upon B, then find 5, the Time upon A, is .3, the Interest of 11. five Years upon B.

K 2

2. Set

2. Set 1 upon A, to 3 upon B, then against 350 the Sum given, upon A, is 105 upon B, that is 105 the Interest of 350 l. in five Years, which was required.

2. For Months.

Exam. What is the Interest of 350 l. for Months, at 6 per Cent. per Ann.?

1. Set 12 upon A, to .06 the Rate upon B, then against 4, the Months given upon A, is .02, the Interest of 1 l. in four Months upon B.

2. Set 1 upon A, to .02 upon B, then against 350 the Sum given upon A, is 7 upon B; that is 7 l. the Interest sought.

3. For Days.

Exam. What is the Interest of 350 l. for seven three days, at 6 per Cent. per Ann.?

1. Set 365 upon A, to .06 the Rate upon B, then against 73 the days given upon A, is .012 the Interest of 1 l. in 73 days upon B.

2. Set 1 upon A, to .012 upon B, then against 350 the Sum given upon A, is 4.2 upon B; that is 4 l. 4 s. the Interest sought.

There are several other useful Questions relating to Simple Interest, some of which shall be resolved the following Problems. In order to which, put

- = A *Principal* or Sum forborn.
 - = The *Time* of Forbearance in Years, or parts of a Year.
 - = The *Gain* or *Rate of Interest*.
 - = The *Amount* of the said Principal, in the said Time, and at the said Rate of Interest.
- Then if any three of these Particulars be given, the fourth may be found.

Problem II.

Given u , g and t : To find a .

$$1 : g \cdot t + 1 :: u : a$$

Q. What will 250 l. amount unto in 6 Years, at 6 per Cent. per Annum?

$u = 250$, $g = .06$, $t = 6$, and:

$$g \cdot t + 1 = 1.36$$

Therefore,

Set 1 upon A, to 1.36 upon B, then against 250 upon A, is 340 upon B, the *Amount* sought.

Problem III.

Given a , g and t : To find u .

$$g \cdot t + 1 :: 1 : a : u$$

Q. What is 340 l. due 6 Years hence worth in ready Money, at 6 per Cent. per Ann.

$$K \ 3$$

$$a =$$

$$a = 350, g = .06, t = 6$$

$$\text{and } g t + 1 = 1.36 \text{ Therefore,}$$

Set 136 upon A, to 1 upon B, then against 3 upon A, is 250 upon B, the *Present Worth* sought.

Problem IV.

Given a, u and g : To find t .

$$u g : 1 :: a - u : t$$

Examp. IN how many Years will 250 l. amount unto 340 l. at 6 per Cent. per Ann.?

$$a = 340, u = 250, g = .06$$

$$u g = 15, \text{ and } a - u = 90.$$

Therefore, Set 15 upon A, to 90 upon B, then against 1 upon A, is 6 upon B, that is, 6 Years Time sought.

Problem III.

P

$$u : a :: 1 : t$$

What is 250 l. due 6 Years hence worth in ready Money at 6 per Cent. per Ann.

Problem V.

Given a , u and t . To find g .

$$ut : 1 :: a - u : g.$$

Exam. **A** What Rate of Interest will 250 l. Amount unto 340 l. in 6 Years?

$$u = 340, a = 250, t = 6$$

$$ut = 1500 \text{ and } a - u = 90$$

Therefore, Set 1500 upon A, to 1 upon B, then 90 upon A, is .06 upon B, which is the Rate of Interest sought.

The Interest of Money for Days, requiring more Exactness than can be expected from any Instrumental Operations, here is added a short Table, which readily shews the Interest of any Sum for any Time, and at any Rate of Interest, to the hundredth part of a Farthing, and is also very useful for other purposes.

K 4

A

A Table of Simple Interest for any Sum, Time, and Rate.

l. s. d. f.				s. d. f.			
1000000	2739	14	6 0.99	90	4	11 0.71	
900000	2469	15	0 3.29	80	4	4 2.41	
800000	2191	15	7 1.59	70	3	10 0.11	
700000	1917	16	1 3.89	60	3	3 1.81	
600000	1643	16	8 2.19	50	2	8 3.51	
500000	1369	17	3 0.49	40	2	2 1.21	
400000	1095	17	9 2.79	30	1	7 2.90	
300000	821	18	4 1.09	20	1	1 0.60	
200000	547	18	10 3.40	10	0	6 2.30	
100000	273	19	5 1.70				
90000	246	11	6 0.33	9	0	5 3.67	
80000	219	3	6 0.96	8	0	5 1.04	
70000	191	15	7 1.59	7	0	4 2.44	
60000	164	7	8 0.22	6	0	3 3.78	
50000	136	19	8 2.85	5	0	3 1.19	
40000	109	11	9 1.48	4	0	2 2.52	
30000	82	3	10 0.11	3	0	1 3.89	
20000	54	15	10 2.74	2	0	1 1.16	
10000	27	7	11 1.37	1	0	0 2.63	
9000	24	13	1 3.23	.900	0	0 2.37	
8000	21	18	4 1.10	.800	0	0 2.10	
7000	19	3	6 2.96	.700	0	0 1.48	
6000	16	8	9 0.82	.600	0	0 1.58	
5000	13	13	11 2.68	.500	0	0 1.32	
4000	10	19	2 0.55	.400	0	0 1.05	
3000	8	4	4 2.41	.300	0	0 0.79	
2000	5	9	7 0.27	.200	0	0 0.53	
1000	2	14	9 2.14	.100	0	0 0.26	
900	2	9	3 3.12	.090	0	0 0.24	
800	2	3	10 0.11	.080	0	0 0.21	
700	1	18	4 1.10	.070	0	0 0.18	
600	1	12	10 2.08	.060	0	0 0.16	
500	1	7	4 3.07	.050	0	0 0.13	
400	1	1	11 0.05	.040	0	0 0.11	
300	0	16	5 1.04	.030	0	0 0.08	
200	0	10	11 2.03	.020	0	0 0.05	
100	0	5	5 3.01	.010	0	0 0.02	

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The use of the Table.

I. To find the Interest of any Sum, for any number of Days, at any Rate of Interest.

Multiply the Sum given by the Rate, and that Product by the number of Days, then set down the Figures of the last Product, with Cyphers towards the right Hand, as in the Example following, the Sums answering to these Numbers taken out of the Table, and added together give the Interest sought.

1. Example. What is the Interest of 6525. l. for 146 Days, at 8 per Cent. per Ann.?

Principal ——— 6525
Rate ——— .08

1. Product ——— 522.00
Number of Days — 146
————— .80
————— .04
————— .3132
————— .2088
————— .522.00 Answer

2. Product ——— 76212

	l.	s.	d.	f.
70000	191	15	7	1. 59
6000	16	8	9	0. 82
200	0	13	11	2. 03
10	0	0	6	2. 30
2	0	0	1	1. 26

Answer ——— 208. 16. 0. 0. 00

K 5

2. Ex.

2. Example. What is the Interest of 825 l. 10 in 146 Days, at 8 per Cent. per Ann.

Principal 825.5
Rate 8

1. Product 6604
Num. of Days 146

39624
28416
6804

2. Product 9641.84

	1.	2.	3.	4.	5.
9000	24.	13.	1.	3.	23
600	7.	12.	10.	2.	08
40	0.	2.	2.	1.	21
1	0.	0.	0.	2.	63
.80	0.	0.	0.	2.	10
.04	0.	0.	0.	0.	11
Answer	26.	28.	3.	3.	36

3. Example. What is the Interest of 824 l. 10 the same Rate, for the Remainder of the Year, viz 219 Days?

1. Product 6604
Number of Days 219

59436
6604
13208

2. Product 14462.76

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	l.	s.	d.	f.
10000 —————	27.	7.	11.	1. 37
4000 —————	10.	19.	2.	0. 55
400 —————	1.	1.	11.	0. 05
60 —————	0.	3.	3.	1. 81
2 —————	0.	0.	1.	1. 26
.70 —————	0.	0.	0.	1. 84
.06 —————	0.	0.	0.	0. 16

The Interest of 825.5 l.
for 219 Days, at 8 per
Cent. is ————— } 39. 12. 5. 3. 04

The Interest of 825.5 l.
for 146 Days, at the
same Rate, was found
to be ————— } 26. 8. 3. 3. 36

The Sum is the exact
Interest of 825.5 l. for
365 Days, or one Year
at 8 per Cent. per Ann.
viz. ————— } 66. 0. 9. 2. 40

II. Any Annual Salary, or Pension, being given, to
find what it amounts to by the Day.

Against any Annual Salary in the first or third
Col. of the Table, you have the Amount for one
Day, in the second or fourth Col. respectively
Thus,

	l.	s.	d.	f.
1000 } Pounds	13.	13.	11.	2. 68
600 } per An.	1.	12.	10.	2. 08
70 } is	0.	3.	10.	0. 11

} per Day.

And

And so for any other Sum, found in the first or third Column of the Table.

And so for any number of Days; multiply the Annual Salary by the number of Days given, and place the Figures of the Product, with Cyphers towards the right Hand, as in the Example following, the Sums answering to these Numbers taken out of the Table, and added together, give the Answer.

Example. At a Salary of \$25 l. per Ann. what is due for 73 Days?

Salary	_____	525
Days	_____	73

1575

3675

38325

30000	_____	82.	3.	10.	0.	11
-------	-------	-----	----	-----	----	----

8000	_____	21.	18.	4.	1.	30
------	-------	-----	-----	----	----	----

300	_____	0.	16.	5.	1.	04
-----	-------	----	-----	----	----	----

20	_____	0.	1.	1.	0.	60
----	-------	----	----	----	----	----

5	_____	0.	0.	3.	1.	15
---	-------	----	----	----	----	----

Answer 105. 0. 0. 0. 00

Of Annuities at Simple Interest.

Problem VI.

To find the Amount of any Annuity for any Time given, and at any Rate of Simple Interest.

A General Rule.

1. Find the Interest of the Annuity for half the number of Years, half Years or Quarters, that the Annuity is to continue.

2. Multiply the Interest thus found by the number of Years, half Years, or Quarters, less by 1, and to this Product add the Sum of the Annual, half yearly, or quarterly Payments, the Sum will be the true Amount of the Annuity.

Example. What is the Amount of an Annuity of 100 l. in 8 Years, at 6 per Cent.?

The Interest of 100 l. for half the time, viz. 4 Years, is 24 l. this multiplied by the number of Years less by 1, (viz. 7.) the Product will be 168, to which add the Sum of the Annual Payments, viz. 800, the Sum will be 968 l. the Amount sought.

Pro;

Problem VII.

To find the present worth of any Annuity for any given Time, at any Rate, accompting Simple Interest.

The Rule, or Proportion is,

As the Amount of 1 l. in any Time is to 1 l.
So is the Amount of any Annuity, to the present worth.

Therefore, Divide the Amount of the Annuity by the Amount of one Pound in the Time given, the Quotient is the present Worth.

Example. What is the present Worth of an Annuity of 100 l. to continue 8 Years, at 6 per Cent.?

The Amount of 100 l. Annuity for 8 Years, by the last Problem, was found to be 968 l. and the Amount to one Pound in 8 Years, will be found (by the II Problem before going) to be 1.48.

Now if 968 (the Amount of the Annuity) be divided by 1.48, the Quotient is 654.056, and so much present Money is equal in Value to an Annuity of 100 l. for 8 years; for if this 654.054 be put out at Interest, it will in 8 Years amount to 968 l. at 6 per Cent. per Ann. Several Writers have treated of the Calculation of Annuities at Simple Interest; but in as much as all present Worths either of Principal Sums, or Annuities due at any Time to come, ought to be consider'd in a Geometrical Proportion, according

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according to the several Rates of *Compound Interest*, all Computations of *Annuities at Simple Interest*, are to be laid aside as useless.

Of Compound Interest.

IN *Compound Interest*, the respective Amounts for each respective Year are so many *Geometrical proportional Numbers*. Thus, if 100 l. be put out to Interest at 6 per Cent. per Annum, there will be due at the end of the first Year 106 l. which 106 is now made a Principal, and at the end of the second Year will amount to 112.36, and this being made a Principal, will at the end of the third Year be increased to be 119.1016; now these four Numbers, viz. 100, 106, 112.36, and 119.1016, are four *Geometrical proportional Numbers*: For,

As 100 to 106, (for the first Year);

So is 106 to 112.36 (for the second Year.)

Again, As 106 to 112.36;

So is 112.36 to 119.1016, for the third Year, &c.

Pro-

Problem VIII.

Any Sum of Money being propounded, to find the Amount thereof in any time, at any Rate of Compound Interest.

Example: What is the Amount of 100 l. being forborn 5 Years, at 6 per Cent. per Ann. Compound Interest?

1. **S**ET 100 upon B, to 106 (the first Year's Amount) upon A; then against 106 upon B, you have 112.36 upon A, the Amount in two years. *In like manner,*

2. Against 112.36 upon B, is 119.1016 upon A, the Amount in three Years. *Again,*

3. Against 119.1016 upon B, is 126.247 upon A, the Amount in four Years.

Lastly, Against 126.247 upon B, you have 133.822 upon A, which is the Amount in 5 Years, and the Answer to the Question.

Problem IX.

A Sum of Money being due at any time to come, to find what it is worth in ready Money, at any Rate of Interest.

Example: What is the present worth of 133.822 l. due at the end of 5 Years to come, at 6 per Cent. per Annum Compound Interest?

THIS is but the Inverse of the last Problem: Therefore if you set the Rule as is there directed, you may read the Proportion back again; For the respective Amounts found upon A, in that Example, will point you to the respective present Worth at the end of each respective Year.

Thus, Set 106 upon A, to 100 upon B; then against 133.822 upon A, is 126.247 upon B: The present Worth of 100 l. at the end of the first Year.

Also, Against 126.247 upon A, you have 119.1016 upon B, the present Worth at the end of the second Year.

Again, Seek 119.1016 upon A, against it you have 112.36 upon B, the present Worth at the end of the third Year.

In like manner, Against 112.36 upon A, is 106 upon B, the present Worth at the end of the fourth Year.

Lastly, Against 106 upon A, you have 100 upon B, which is the present Worth of 133.822 l. at the end of five Years, which was required.

Pro-

Problem X.

A yearly Rent or Annuity being forborn a certain number of Years, to find what the Arrearages thereof will amount unto at any Rate propounded.

1. Find a Sum (or Principal) whose Interest is equal to the Annuity or yearly Rent.

2. Find (by *Probl. VIII.*) the Amount of that Sum in the time given; This done, subtract the Principal first found from the Amount, the Remainder will be the Answer.

Example: A Rent or Annuity of 9 L. per Annum being forborn 22 Years, what will the Arrearages thereof amount unto, accompting Interest upon Interest, at 6 per Cent.?

To find a Principal whose Interest is 9 l. say,

As 6 to 100;

So is 9 to 150, the Principal sought.

The Amount of 150 l. in 22 Years, will be found by Problem VIII. to be 301.828, from which subtract (the Principal first found, viz.) 150, the Remainder will be 151.828, the Sum of the Arrearages sought.

Pro-

Problem XI.

A yearly Rent or Annuity being propounded, to find what it is worth in ready Money?

First, find (by the last Problem) what the Arrearages thereof will amount unto at the end of the Term propounded: Then (by Probl IX.) find the present Worth of those Arrearages: The present Worth thus found is the value of the Annuity or Yearly Rent sought.

Example: Suppose a Rent of 9 l. per Ann. to continue 12 Years, What Sum of Money may be given for this Rent, allowing the Buyer 6 per Cent. Compound Interest?

By the last Problem the Arrearages of a Rent of 9 l. per Ann. being forborn 12 Years, were found to be 151.828 l.

And the present Worth of 151.828, due at the end of 12 Years to come, will be found (by Probl IX.) to be 75.443, and so much may be given for a Rent of 9 l. per Ann. to continue 12 Years.

But suppose the Term of the Rent or Annuity begins not till after the Expiration of certain Years to come, in such case, find what the Arrearages forborn for all that time are worth in ready Money, for that is the Answer.

For

For Instance.

Suppose in the last Example, the Annuity of 9 l. for 12 Years, were not to begin till after the Expiration of 6 Years: I here seek what the Arrearages, (*viz.* 151.828) being forborn 18 Years are worth in ready Money, and by *Problem IX.* I find it will be 53.185 l. which is the Answer: So a Yearly Rent or Annuity of 9 l. *per Annum*, to begin 6 Years hence, and continue 12 Years, is worth but 53.185, in ready Money.

Problem XII.

A Sum of Money being propounded, to find what Annuity (to continue any Number of Years, and at any Rate proposed) that Sum will buy.

TAKE any Annuity at pleasure, and find the Value thereof in ready Money: This done, the Proportion is,

*As the present Worth found, is to the Annuity taken;
So is the Sum given, to the Annuity sought.*

Example: *What Annuity to continue 12 Years will 300 l. Purchase, allowing the Buyer 6 per Cent. Compound Interest?*

By *Problem XI.* I find that 75.443 will purchase 9 l. *per Annum*, to continue 12 Years.

Therefore, set 75.443 upon B, to 9 upon A; then again 300 upon B, is 35.776 upon A, that is 35 l. 15 s. 6 d.; and so much Annuity to continue 12 Years is worth 300 l.

Pro-

Problem XIII.

There is a Freehold Estate of the yearly Value of 78 l. What Sum of Money is this Estate worth, allowing the Buyer 6 per Cent. per Ann. Compound Interest for his Money?

The Rule or Proportion is,

As the Annual, half Yearly or Quarterly Interest of 1 l. is to 1 l.;

So is the Annual, half Yearly, or Quarterly Rent to the Sum sought.

The Interest of 1 l. at 6 per Cent. Compound Interest,

$$\text{For } 1 \left\{ \begin{array}{l} \text{Year} \\ \text{Half Year} \\ \text{Quarter} \end{array} \right\} \text{ is } \left\{ \begin{array}{l} .06 \\ .02956 \\ .01467 \end{array} \right\}$$

Now to Answer the Question.

Set .06 upon B, to 1 upon A; then against 78 upon B, is 1300 upon A, which is the Value of the Estate propounded; and for the same reason I find without moving the Rule, that a Freehold Estate

$$\text{of } \left\{ \begin{array}{l} 90 \\ 120 \\ 150 \end{array} \right\} \text{ Pounds } \left\{ \begin{array}{l} 1500 \\ 2000 \\ 2500 \end{array} \right\} \text{ Pounds.}$$

per Ann. is worth

But

But suppose the Rent of the former Estate, (*viz.* 78 *l.*) were payable *Quarterly*, (*viz.* 19.5 *l.* per Quarter :) *What it is then worth*, allowing the Purchaser 6 per Cent. as before?

Set .01467 (the Interest of 1 *l.* for a Quarter of a Year) upon A, to 1 upon B; then against 19.5 (the Quarter's Rent) upon A, you have 1329.244 upon B, that is 1329 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* the Answer, which is 29 *l.* 4 *s.* 10 $\frac{1}{2}$ *d.* more than the former; that being accounted by Yearly, this by Quarterly Payments.

Problem XIV.

A Sum of Money propounded, to find the Annual Rent of a Free-hold Estate that such a Sum will purchase, at a given Rate of Compound Interest.

This is the Inverse of the last, and therefore the Proportion is:

As 1 to the *Annual Interest*;

So is the Sum propounded to the *Annual Rent*.

*Example: Suppose I would lay out 3650 *l.* upon a Free-hold Estate, so as to have 6 per Cent. Comp. Interest for my Money, What must the Annual Rent be?*

Set 1 upon A, to .06 upon B; then against 3650 upon A, you have 219 upon B: The Answer is 219 *l.* per Annum.

Pro-

Problem XV.

The Annual Rent of a Freehold Estate being known, and also the Sum of Money which was paid for the same, to find what Rate of Compound Interest the Purchaser shall have for his Money.

The Proportion is,

As the Sum given is to 1 ;
So is the Annual Rent to the Rate of Interest.

Example : There is 1300 l. given for a Freehold Estate, whose Annual Rent is 78 l. What Rate of Interest has the Purchaser for his Money?

Set 1300 upon A, to 1 upon B ; then against 78 upon A, is .06 upon B : So the Purchaser shall have 6 per Cent. for his Money.

If it be demanded, How many Tears Purchase a Freehold Estate is worth at any Rate of Compound Interest ?

The Proportion is,

As the Rate of Interest is to 1 ;
So is 1 to the Answer.

Example, at 5 per Cent.

Set .05 upon B, to 1 upon A ; then against 1 upon B, is 20 upon A ; so at 5 per Cent. it is worth 20 Tears Purchase.

At

At 6 per Cent. it is worth but 16.66 Years Purchase. For,

As .06 upon B, to 1 upon A;

So is 1 upon B, to 16.66 upon A; that is, 16 Years
8 Months, and somewhat more.

The Converse of this is, to find by the Number
Years Purchase, what Rate of Interest the Buyer has
for his Money.

Say, As the Number of Years Purchase is to 1;
So is 1 to the Rate of Interest.

Example: A Freehold Estate is bought at 20 Years
Purchase; What Rate of Interest has the Buyer
for his Money?

Set 20 upon A, to 1 upon B, then against 1 upon
A, is .05 upon B; that is, 5 per Cent. per Ann.

SECT. XIII.

The Rule of Fellowship.

Problem I.

Five Persons C D E F and G, make a Bank,
or Stock of Money, in all 360 l. with which
they Trade and gain 40 l. what part of this
Gain must each Person receive to balance his
Money put in?

THE Instrument will resolve this and such like
Questions by Inspection.

The Proportion is,

As the whole Stock,
is to the whole Gain;

So is each Man's particular Stock,
to his particular Gain.

Therefore, Set 360 the whole Stock upon A, to 40
the whole Gain upon B, then against each Person's
stock upon A, you have his Gain upon B. Thus
against 108 upon A, you have 12 upon B. So C
who put 108 l. shall have 12 l. of the Gain, and so
the rest.

L

Stock

	Stock.	Gain.
C	108	12
D	81	9
E	63	7
F	72	8
G	36	4
	360	40

By which it appears, that this Question (which by the Pen would require five Operations) is resolved by the Rule at one.

A Question of this Rule with Time.

Problem II.

Two Merchants join together, H put in 60 for 6 Months, F put in 80 l. for 8 Months and they gained 50 l.; How much must each Merchant have of the Profit?

The Proportion will be,

(1) As 1 is to 6 (Months);

So is 60 l. to 360.

(2) As 1 is to 8 (Months);

So is 80 l. to 640.

Set 1 upon A, to 6 upon B; then against 60 upon A, is 360 upon B.

Again

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Again, Set 1 upon A, to 8 upon B; then against 640 upon A, is 640 upon B, the Sum of these is 1000.

Now for each Merchant's Share.

1. Set 1000 upon A, to 50 upon B; then against 360 upon A, is 18 upon B, that is 18 l., and so much is H's Share.

2. Set 1000 upon A, to 50 upon B; then against 640 upon A, is 32 upon B, which is the Share belonging to F.

So that $\left\{ \begin{matrix} H \\ F \end{matrix} \right\}$ must have $\left\{ \begin{matrix} 18 l. \\ 32 l. \end{matrix} \right.$

The whole Gain 50 l.

Example. L. 2. **SECT.**

SECT. XIV.

The Use of the RULE

IN
Military AFFAIRS

Problem I.

Any number of Men being proposed, to place them in Battalia, so that there may be many in Rank as in File, and supposing them to be imbattelled at Order (which is 3 Rank distance in Rank and File) to find how many Square Foot of Ground they will take up?

Example. **L**ET the Number of Men proposed be 3600. Set the Lines C and D equal at the end. Now forasmuch as the Number given doth consist of an even Number of Figures seek the Number, viz. 3600 in the second Radix on the Line C, and against it on the Line D,

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will find 60, and so many Men must be in Rank, and as many in File :

Now to find how many square Foot of Ground they will occupy :

Set 1 upon A to 3 upon B, then against 60 upon A is 180 upon B.

Then set the Lines C and D even, and find this Number, viz. 180 on the first Radius on the Line D, and against it you have 32400 upon C, and so many square Foot of Ground will these Soldiers occupy, being at their Order of three Foot.

Problem II.

Any Number of Men being proposed, to place them in Battalia, in such order that there may be twice as many in Rank as in File, and that they may stand at close Order, which is $1\frac{1}{2}$ Foot.

Example. **L**ET the Number given be 1800, set the Lines D and C even, and count half the Number given, viz. 900 upon the Line C, and against it you have 30 upon D, which is the Depth of the File, and then there must be 60 in Rank, which is twice 30.

Now for the Ground these will occupy being at close Order.

Set 1 upon A to 1.5 upon B, then against 30 upon A is 45 the side of the Ground, _____ and without moving the Lines, seek 60 upon the Line
L 3. A,

A, and against it you have 90 upon B, which is the Length of the Ground.

Now for the Area.

Set 1 upon A, to 45 upon B; then against 90 upon A, is 4050, and so many square Foot of Ground will these 1800 Men occupy at Close Order.

Problem III.

Any Number of Men being proposed to be put in Battalia, and a certain Number named to be either in Rank or File, to find the other Number.

Example. **L**ET it be required to place 1200 Men in Battalia, so that there shall be 10 in File, how many must there be in Rank; or how many Files must there be?

This is resolv'd by the Lines A and B. And the Proportion is,

As 15 the Depth in File, is to 1;

So is 1200, the Number proposed, to 80 the Number of Men in Rank.

Pro-

Problem IV.

Any Number of Soldiers being given, together with their distance in Rank and File, to order them into a Square Battalia of Ground.

Example. **L** E T the Number of Soldiers given be 5400, their Distance in File 6 Foot, and in Rank 4 Foot.

Set 6 upon the Line A, to 4 upon the Line B, and then against 5400 upon A, is 3600 upon B.

Then set the Lines D and C even, and seek 3600 on the second Radius upon C, against it you have 60 upon D, the number of Men to be placed in File.

Now for the number of Men in Rank, say

As 60 is to 1;
So is 5400 to 90.

So there shall be 60 in File, and 90 in Rank.

Now if you multiply 60, the Number of Men in File, by 6, their Distance, the Product will be 360, the side of your Ground.

In like manner, if you multiply 90, the Number of Men in Rank, by 4, their Distance, the Product will be 360, as above; which shews, your Ground exactly square, and is therefore a Proof of the Work.

L 4.

Pro²

Problem V.

If 1000 Soldiers may be Lodged or Quartered in a square Piece of Ground, whose side is 400 Foot; how many Foot long must the side of a Square be, that the Ground included may lodge 4000?

The Proportion will be

- (1) **A**s 1 is to 400 (the Side of the Square which will Lodge 1000 Soldiers)
So is 4000, to 160000.
- (2) **A**s 1000 is to 4000;
So is 160000 to 640000.

Now (setting the Lines D and C even) find the 640000 in the second Radius on the Line G, and against it you have 800 upon D, and so much must the Side of a Square be, to Lodge 4000 Soldiers with the same convenience that a 1000 were Lodged in a Square whose Side was 400 Foot.

SECT

SECT. XV.

Of Triplicate Proportion.

Triplicate Proportion, is such a Proportion as is between Lines and Solids, or between Solids and Lines, of which I shall give some Examples in the following Problems.

Problem I.

If a Bullet whose Diameter is 4 Inches, weighs 9 Pounds, what shall another Bullet of the same Metal weigh, whose Diameter is 8 Inches?

SET 4 (the Diameter of the Bullet proposed) up on the single Line, to 9 the weight on the Line E, then against 8 upon the single Line is 72, which is the weight of the Bullet whose Diameter is 8 Inches.

And as the Lines now stand, you may find the weight of all Bullets of the same Metal; for against the Diameter upon the single Line you have the weight upon the Line E.

L 5

Thus

Thus, if the Diameter of a Bullet

$$\begin{array}{l} \text{be } \left\{ \begin{array}{l} 3 \\ 5 \\ 6 \\ 7 \end{array} \right\} \text{ Inches, the} \\ \text{weight will be } \left\{ \begin{array}{l} 4.79 \\ 17.17 \\ 30.37 \\ 48.23 \end{array} \right\} \text{ Pounds.} \end{array}$$

Problem II.

Having the Weight of two Bullets of the same Metal, and the Diameter of one of them, to find the Diameter of the other.

THIS is but the *Converse* of the last, and therefore without moving the Lines, find the Weight on the Line E, and against it you have the Diameter on the single Line.

Thus, if a Bullet

$$\begin{array}{l} \text{weighs } \left\{ \begin{array}{l} 20 \\ 30 \\ 40 \end{array} \right\} \text{ Pounds, the} \\ \text{Diameter will be } \left\{ \begin{array}{l} 5.23 \\ 5.97 \\ 6.67 \end{array} \right\} \text{ Inches.} \end{array}$$

Problem

Problem III.

If a Brass Gun, whose Diameter is 8 Inches, weighs 640 Pounds, what shall another Gun weigh (of the same Shape and Metal) whose Diameter is 6 Inches?

SET 8 upon the single Line, to 640 on the Line E, then against 6 upon the single Line, is 270 on the Line E, which is the Weight of the Piece whose Diameter is 6 Inches.

Problem IV.

There is a Conical Tun which contains 620 Gallons, the Diameter at the Bottom is 80 Inches, the Diameter at the Top 71.2 Inches, and the Depth 30 Inches; it is desired to make another Tun in the same Shape with the former, that may contain 800 Gallons. To find the Diameter of this Tun at the Bottom and Top, and also the Depth.

1. For the Diameter at the Bottom.

SET 620 (the Content of the first Tun) upon the Line E, to 80 (the Bottom-Diameter) on the single Line: Then against 800 (the Content of the second Tun) upon E, is 87, which is the Diameter sought.

2. For

2. For the Diameter at the Top.

Set 620 upon E to 71.2 upon the single Line then against 800 upon E is 77.4 the Diameter sought.

3. For the Depth.

Set 620 upon E to 30 upon the single Line, then against 800 upon E is 32.63 the Depth required.

So the Diameters and Depth of this Tun are as followeth,

$$\begin{array}{l} \text{Bottom} \\ \text{Top} - \end{array} \left. \vphantom{\begin{array}{l} \text{Bottom} \\ \text{Top} - \end{array}} \right\} \begin{array}{l} \text{Diameter} \\ \text{Depth} \end{array} \left\{ \begin{array}{l} 87 \\ 77.4 \\ 32.63 \end{array} \right\} \text{Inches.}$$

But the first of these being found as above directed, the other may be discover'd at one Operation by the Lines A and B. Thus:

Set 80 upon A to 87 upon B, then against 71.2 upon A is 77.4 upon B, and against 30 upon A is 32.63 upon B, which also shews, that the Diameter and Depth of this Tun, are in proportion to the Diameters and Depth of the Tun first proposed. And the like may be done for any Casks: of which need not give any Examples.

Problem

Problem V.

There is a Ship of 60 Tun Burthen, which measures 44 Foot by the Keel, and it is required to make another Ship of 360 Tun Burthen, of the same Shape with the first, How many Foot long must the Keel of the second Ship be?

SET 60 upon E, to 44 upon the single Line, then against 360 upon E, is 80 on the single Line, which is the Length of the Keel required.

Now suppose in the first Ship the middle Beam be 17.6 Foot, and the Depth of the Hold 7.76 Foot.

To find these Dimensions of the second Ship,

Set 44 upon A, to 80 upon B, then against 17.6 upon A, is 32 upon B, which is the Length of the Beam. And without moving the Lines, seek 7.76 upon A, and against it you have 14.1 upon B, which is the Depth of the Hold. So the Dimensions and Burthen of the two Ships are as followeth,

	1 Ship	2 Ship	
Length of the { Keel }	44	80	} Foot.
{ Beam }	17.6	32	
Depth of the Hold —	7.76	14.1	
Burthen —	60	360	Tun.

To

To find the Burthen of any Ship, whose Length, Breadth, and Depth is given in Feet, the Common Rule is,

Multiply the Length of the Keel by the Length of the Midship Beam, then multiply that Product by the Depth of the Hold; then divide this last Product by 100, the Quotient gives you the Tuns. But when there is no Allowance for Guns, divide the last Product by 95.

Thus, in the second Ship abovementioned, multiply 80, the Length, by 32 the Breadth, the Product is 2560; this multiplied by 14.1, the Depth, gives 36096; and this divided by 100, is 360.96, that is, 360 Tons, and 96 parts of a Ton, which is the Burthen of this Ship, allowing for Guns. But if there be no Allowance made for Guns, divide the last Product, *viz.* 360.96, by 95, in this case her Burthen will be found to be 379.95, that is, 380 Ton very near.

SECT.

S E C T. XVI.

P R O P O R T I O N S

I N A

C I R C L E.

IT was shewn in Page 42, that if the *Diameter* of a Circle be 1, the *Circumference* will be 3.141592, and the *Area* .785398, the Square-Root of .785398 is .886227, and this is the Side of a Square, equal to the Area of a Circle, whose Diameter is 1. And the Square-Root of .5, viz. .707107 is the Side of the greatest Square that can be inscribed in a Circle, whose Diameter is 1: So that if the Diameter of a Circle be 1,

$$\text{the } \left\{ \begin{array}{l} \text{Circumference} \\ \text{Square equal} \\ \text{Squ. inscribed} \\ \text{Area} \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 3.141593 \\ .886227 \\ .707107 \\ .785398 \end{array} \right\}$$

[Note,

Note further, that if the Circumference of Circle be 1,

$$\text{The } \left\{ \begin{array}{l} \text{Diameter} \\ \text{Sq. equal} \\ \text{Sq. Inscribed} \\ \text{Area} \end{array} \right\} \text{ will be } \left\{ \begin{array}{l} .318310 \\ .282095 \\ .225079 \\ .079577 \end{array} \right\}$$

If the Area of a Circle be 1.

$$\text{The } \left\{ \begin{array}{l} \text{Diameter} \\ \text{Circumference} \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 1.128379, \\ 3.54491, \end{array} \right\}$$

If the Circumference of a Circle be 4,

$$\text{The } \left\{ \begin{array}{l} \text{Diameter} \\ \text{Area} \end{array} \right\} \text{ is } 1.27324,$$

If the Diameter of a Circle be 4,

$$\text{The } \left\{ \begin{array}{l} \text{Circumference} \\ \text{Area} \end{array} \right\} \text{ is } 12.56637.$$

In all these Cases the *Area* is Square Inches, Foot Yards, &c. the other Terms being Lines, do relate to Lineal, Inches, Foot, Yards, &c. respectively. These Things premised, I shall here add some useful Questions, which may be resolved either by Pen or Instrument.

1. *The Diameter of a Circle given, to find the Circumference.*

As 1 is to 3.141593, Or

As .31731 is to 1 ;

So is the Diameter to the Circumference.

2. *The Circumference given, to find the Diameter.*

As 1 is to .31831, Or

As 3.141593 is to 1 ;

So is the Circumference to the Diameter.

3. *The Diameter given, to find the Side of a Square, equal to the Circle.*

As 1 is to .886227, Or

As 1.128379 is to 1 ;

So is the Diameter to the Side of a Square equal.

4. *The Side of a Square given, to find the Diameter of a Circle whose Area is equal to the Square.*

As 1 is to 1.128379, Or

As .886227 is to 1 ;

So is the side of the Square to the Diameter sought.

5. *The Circumference given, to find the side of a Square equal to the Circle.*

As 1 is to .282095, Or

As 3.54491 is to 1 ;

So is the Circumference to the side of the Square required.

6. *The*

6. The Side of a Square given, to find the Circumference of a Circle equal to the Square.

As 1 is to 3.54491, Or,

As .282095, is to 1 ;

So is the Side of the Square to the Circumference sought.

7. The Diameter of a Circle given, to find the Side of the greatest Square inscribed.

As 1 is to .707107, Or

As 1.41421 is to 1 ;

So is the Diameter to the Side required.

8. The Side of a Square given, to find the Diameter of a Circle that shall circumscribe the Square.

As 1 is to 1.41421, Or

As .707107 is to 1 ;

So is the Side of the Square to the Diameter sought.

9. The Circumference of a Circle given, to find the Side of the Square inscribed.

As 1 is to .225079, Or

As 4.4429 is to 1 ;

So is the Circumference to the Side required.

10. *The Side of a Square given, to find the Circumference of a Circle that shall circumscribe the Square.*

As 1 is to 4.4429, Or

As 225079 is to 1;

So is the Side of the Square to the Circumference sought.

11. *The Diameter of a Circle given to find the Area.*

As 1 is to .785398, Or

As 1.27324 is to 1;

So is the Square of the Diameter to the Area required.

12. *The Area of a Circle given, to find the Diameter.*

As 1 is to 1.27324, Or

As .785398 is to 1;

So is the Area to the Square of the Diameter, whose Square-Root is the Diameter sought.

13. *The Circumference of a Circle given, to find the Area.*

As 1 is to .079577, Or

As 12.56636 is to 1;

So is the Square of the Circumference to the Area sought.

14. *The Area of a Circle given, to find the Circumference.*

As 1 is to 12.56637, Or

As .079577 is to 1;

So is the Area to the Square of the Circumference, whose Square-Root is the Circumference required.

Such of these Proportions as relate to the finding the Area of a Circle by the Diameter or Circumference, being of most general use, and requiring, in some cases, more exactness than can be attained to by the Numbers here given, I have here added two Tables for that purpose, which were Calculated to sixteen Places, and are here Printed exactly true to ten Places in the Fractions, only the last Figure is made one more, where the eleventh Figure was above five.

A Table shewing the Area and Circumference of any Circle in Inches, Foot, Yards, &c. to any Diameter in Inches, Foot, Yards, &c. from 1 to 12, or from 10 to 120.

Dia.	Area.	Circumference.
1.0	7853981634	3.1415926536
1.1	9503317777	3.4557519189
1.2	1.1809733553	3.7699111843
1.3	1.3273228961	4.0840704497
1.4	1.5393804003	4.3982297150
1.5	1.7671458676	4.7123889804
1.6	2.0106192983	5.0265482457
1.7	2.2698006922	5.3407075111
1.8	2.5446900494	5.6548667765
1.9	2.8352873699	5.9690260418
2.0	3.1415926536	6.2831853072
2.1	3.4636059006	6.5973445725
2.2	3.8013271108	6.9115038379
2.3	4.1547562844	7.2256631033
2.4	4.5238934212	7.5398223686
2.5	4.9087385212	7.8539816340
2.6	5.3092915846	8.1681408993
2.7	5.7255526112	8.4823001647
2.8	6.1575216010	8.7964594301
2.9	6.6051985542	9.1106186954
3.0	7.0685824706	9.4247770608

Dia.	Area.	Circumference.
3.1	7.5476763502	9.7389372261
3.2	8.0424771932	10.0530964915
3.3	8.5529859994	10.3672557568
3.4	9.0792027689	10.6814150222
3.5	9.6211275016	10.9955742876
3.6	10.1787601976	11.3097335529
3.7	10.7521008569	11.6238928183
3.8	11.3411494795	11.9380520836
3.9	11.9459060653	12.2522013490
4.0	11.5663706144	12.5663706144
4.1	13.2025431267	12.8805298797
4.2	13.8544236023	13.1946891451
4.3	14.5220120412	13.5088484104
4.4	15.2053084423	13.8230076758
4.5	15.9043128088	14.1371669412
4.6	16.6190251375	14.4513262065
4.7	17.3494454294	14.7654854719
4.8	18.0955736847	15.0796447372
4.9	18.8574099032	15.3938040026
5.0	19.6349540849	15.7079632679
5.1	20.4282062300	16.0221225333
5.2	21.2371663383	16.3362817987
5.3	22.0618344098	16.6504410640
5.4	22.9022104447	16.9646003294
5.5	23.7582944428	17.2787595947
5.6	24.6300864041	17.5929188601
5.7	25.5175863288	17.9070781255
5.8	26.4207942167	18.2212373908
5.9	27.3397100679	18.5353966562
6.0	28.2743338823	18.8495559215

Dia.	Area.	Circumference.
6.1	29.2246656600	19.1637151869
6.2	30.1907054010	19.4778744523
6.3	31.1724531052	19.7920337176
6.4	32.1699087728	20.1061929830
6.5	33.1830724035	20.4203522483
6.6	34.2119439976	20.7345115137
6.7	35.2565235549	21.0486707791
6.8	36.3168110755	21.3628300444
6.9	37.3928065594	21.6769893098
7.0	38.4845100065	21.9911485751
7.1	39.5919214169	22.3053078405
7.2	40.7150407905	22.6194671058
7.3	41.8538681274	22.9336263712
7.4	43.0084034276	23.2477856366
7.5	44.1786466911	23.5619449019
7.6	45.3645979178	23.8761041673
7.7	46.5662571078	24.1902634326
7.8	47.7836242611	24.5044226980
7.9	49.0166993776	24.8185819734
8.0	50.2654824574	25.1327412287
8.1	51.5299735005	25.4469004941
8.2	52.8101725068	25.7610597594
8.3	54.1060794765	26.0752190248
8.4	55.4176944093	26.3893782902
8.5	56.7450173055	26.7035375555
8.6	58.0880481649	27.0176968209
8.7	59.4467869876	27.3318560862
8.8	60.8212337735	27.6460153516
8.9	62.2113885227	27.9601746169
9.0	63.6172512352	28.2743338823

Dia.	Area.	Circumterence.
9.1	65.0388219109	28.5884931477
9.2	66.4761005500	28.9026524130
9.3	67.9290871522	29.2168116784
9.4	69.3977817178	29.5309709437
9.5	70.8821842466	29.8451302091
9.6	72.3822947387	30.1592894745
9.7	73.8981131941	30.4734487398
9.8	75.4296396127	30.7876080052
9.9	76.9768739946	31.1017672705
10.0	78.5398163397	31.4159265359
10.1	80.1184666482	31.7300858013
10.2	81.7128249199	32.0442450666
10.3	83.3228911548	32.3584043320
10.4	84.9486653531	32.6725635973
10.5	86.5901475146	32.9867228627
10.6	88.2473376393	33.3008821281
10.7	89.9202357274	33.6150413934
10.8	91.6088417787	33.9292006588
10.9	93.3131557933	34.2433599241
10.0	95.0331777711	34.5575191895
11.1	96.7689077122	34.8716784548
11.2	98.5203456166	35.1858377202
11.3	100.2874914842	35.4999969856
11.4	102.0703453151	35.8141562509
11.5	103.8689071093	36.1283155163
11.6	105.6831768668	36.4424747816
11.7	107.5131545875	36.7566340470
11.8	109.3588402715	37.0707933124
11.9	111.2202339187	37.3849525777
12.0	113.0973355292	37.6991118431

The Use of the foregoing Table is obvious, for

If the Diameter of a Circle be *Inches, Feet, Yards, &c.* not exceeding 12, you have it in the first Column to every tenth part of an *Inch, Foot, Yard, &c.* And against any Diameter in the first Column, you have the Area in the second Column, and the Circumference in the third: And the Area is always in *Inches, Feet, Yards, &c.* according as the Diameter or Circumference is given in *Inches, Feet, Yards, &c.*

If the Diameter given be entire *Inches, Feet, Yards, &c.* not exceeding 120, you have it in the first Column, by taking away the Point, for by this means the Tenths are made Units; but in this case you must remove the Point two places towards the right Hand in the Area, and one place in the Circumference.

Thus if the Diameter

11.6 the { Area is 105.6831768668
Circumfer. is 36.4424747816

But if the Diameter

116, the { Area is 10568.31768668
Circumfer. is 364.424747816

And for the same Reason if the Diameter

1160, the { Area is 1056831.768668
Circumfer. is 3644.24747816

3. If any Area in the Table be multiplied by 4, the Product will be the Area of a Circle of twice the Diameter of the Circle given, and therefore the Area of a Circle whose Diameter is 22.6 (though not found in the Table) may be readily obtained, for the half of 22.6 is 11.3; which found in the first Column, the Area against it is 100.287491842, this multiplied by 4, is 401.1499659368, the Area of a Circle whose Diameter is 22.6,

4. If the Diameter and Altitude of a *Cylinder* or *Cone* be given, multiply the Area against the Diameter by the Altitude, the Product will be the Content of the *Cylinder*; or by one third part of the Altitude, the Product will be the Content of the *Cone*.

5. If a *Cylinder*, *Sphere* and *Cone*, have the same Diameter and Altitude, their Contents are as 3, 2, 1. Suppose the Diameter and Altitude 12, the Area against 12 is 113.0973355292, this multiplied by

$$\left. \begin{array}{l} 12 \\ 8 \\ 4 \end{array} \right\} \text{ gives } \left\{ \begin{array}{l} 1357.1680263504 \\ 904.7786842336 \\ 452.3893421168 \end{array} \right\} \begin{array}{l} \text{Cylinder.} \\ \text{Sphere.} \\ \text{Cone.} \end{array}$$

So that any Area multiplied by two thirds of its Diameter, gives the Solidity of a *Sphere* of that Diameter.

6. If the Diameter of a *Sphere* be doubled, the Area against the Sum is the Superficial Content of the *Sphere*. As let the Diameter be 6, the double is 12, the Area against 12, is 113.0973355292, the superficial Content of a *Sphere* whose Diameter is 6.

A TABLE shewing the *Area* and *Diameter* of any Circle, in *Inches*, *Foot*, *Yards*, &c. to any *Circumference* in *Inches*, *Foot*, *Yards*, &c. from 1 to 12 or from 10 to 120.

Circ.	Area.	Diameter.
1.0	.0795774715	.5183098862
1.1	.0962887406	.5501408748
1.2	.1145913590	.5819718634
1.3	.1344859269	.6138028520
1.4	.1559718442	.6456338407
1.5	.1790193110	.6774648293
1.6	.2037183272	.5092958179
1.7	.2299788928	.5411268065
1.8	.2578310078	.5729577951
1.9	.2872746723	.6047887837
2.0	.3183098862	.6366197724
2.1	.3509366495	.6684507610
2.2	.3851549623	.7002817496
2.3	.4209648245	.7321127382
2.4	.4583662361	.7639437268
2.5	.4973591972	.7957747155
2.6	.5379437977	.8276057041
2.7	.5801197676	.8594366927
2.8	.6238873769	.8912676818
2.9	.6692465357	.9230986699
3.0	.7161972439	.9549296586

Circ.	Area.	Diameter.
3.1	.7647395016	.9867606472
3.2	.8148733086	1.0185916358
3.3	.8665986651	1.0504226244
3.4	.9199155711	1.0822536130
3.5	.9748240264	1.1140846016
3.6	1.0313240312	1.1459155903
3.7	1.0894155855	1.1777465789
3.8	1.1490986891	1.2095775675
3.9	1.2103733422	1.2414085561
4.0	1.2732395447	1.2732395447
4.1	1.3376972967	1.3050705333
4.2	1.4037465981	1.3369015220
4.3	1.4713874489	1.3687325106
4.4	1.5406198491	1.4005634992
4.5	1.6114437988	1.4323944878
4.6	1.6838592979	1.4642254764
4.7	1.7578663464	1.4960564651
4.8	1.8334649444	1.5278874537
4.9	1.9106550918	1.5597184423
5.0	1.9894367886	1.5915494309
5.1	2.0698100349	1.6233801495
5.2	2.1517748306	1.6552114082
5.3	2.2353318757	1.6870423968
5.4	2.3204790703	1.7188733854
5.5	2.4072185143	1.7507043740
5.6	2.4955495077	1.7825353626
5.7	2.5854720505	1.8143663512
5.8	2.6769861428	1.8461973399
5.9	2.7700917845	1.8780283285
6.0	2.8647889757	1.9098593171

Circ.	Area.	Diameter.
6.1	2.9610777162	1.9416901057
6.2	3.0589580062	1.9735212943
6.3	3.1584298457	2.0053522830
6.4	3.2594932346	2.0371832716
6.5	3.3621481728	2.0690142602
6.6	3.4663946655	2.1008452488
6.7	3.5722326977	2.1326762374
6.8	3.6796622843	2.1645072260
6.9	3.7886834263	2.1963382147
7.0	3.8992961058	2.2281692033
7.1	4.0115003406	2.2600001919
7.2	4.1252961249	2.2918311805
7.3	4.2406834587	2.3236621691
7.4	4.3576623419	2.3554931578
7.5	4.4762327745	2.3873241464
7.6	4.5963947565	2.4191551350
7.7	4.7181482880	2.4509861236
7.8	4.8414939689	2.4828171122
7.9	4.9664299992	2.5146481009
8.0	4.0929581789	2.5464790895
8.1	5.2210779081	2.5783100781
8.2	5.3507891867	2.6101410667
8.3	5.4820920148	2.6419720553
8.4	5.6149863923	2.6738030439
8.5	5.7494728192	2.7056340326
8.6	5.8855497955	2.7374650212
8.7	6.0232188213	2.7692960093
8.8	6.1624793965	2.8011269984
8.9	6.3033315212	2.8329579870
9.0	6.4457751952	2.8647889757

Circ.	Area.	Diameter.
9.1	6.5898104187	2.8966109643
9.2	6.7354371916	2.9234509529
9.3	6.8826555140	2.9602819415
9.4	7.0314653858	2.9921129301
9.5	7.1818668070	3.0239439187
9.6	7.3338597777	3.0557749074
9.7	7.4874442978	3.0876058960
9.8	7.6426203673	3.1194358846
9.9	7.7993879862	3.1512678732
10.0	7.9577471546	3.1830988618
10.1	8.1176978724	3.2149298505
10.2	8.2792401396	3.2467608391
10.3	8.4423739563	3.2785918277
10.4	8.6070993224	3.3104228163
10.5	8.7734162379	3.3422538049
10.6	8.9413247029	3.3740847935
10.7	9.1108247153	3.4059157822
10.8	9.2819162811	3.4377467708
10.9	9.4545993944	3.4695777594
11.0	9.6288740571	3.5014087480
11.1	9.8047402692	3.5332397366
11.2	9.9821980307	3.5650707253
11.3	10.1612473417	3.5969017139
11.4	10.3418882021	3.6287327025
11.5	10.5241206120	3.6605636911
11.6	10.7079445712	3.6923946797
11.7	10.8933600799	3.7242256684
11.8	11.0803671381	3.7560566570
11.9	11.2689657456	3.7878876456
12.0	11.4591559026	3.8197186342

The Uses of the Table are very plain, for

1. IF the *Circumference* of a Circle be given in *Inches, Foot, Yards, &c.* not exceeding 12, you have it in the first Column to every tenth part of an *Inch, Foot, Yard, &c.* from 1, to 12, and from 10, to 120, in Intire *Inches, Foot, Yards, &c.* and in either case you have the Area in the second Column, and the Diameter in the third Column,

Thus if the *Circumference* of a Circle

be 10.5, the $\begin{cases} \text{Area is } 8.7734162379 \\ \text{Diam. is } 3.3422538049 \end{cases}$

But if the *Circumference*

be 105, the $\begin{cases} \text{Area is } 877.34162379 \\ \text{Diam. is } 33.422538049 \end{cases}$

2. If any Area in this Table be multiplied by 4, you have the Area of a Circle to twice the *Circumference* of the Circle given. Let 8.9413247029 (the Area against 10.6 *Circumference*) be multiplied by 4, the Product will be 35.7652988116 which is the Area of a Circle whose *Circumference* is 21.2.

3. If the *Circumference* of a *Cylinder* be 10.6 Foot, and the *Altitude* 16 Foot, the Area against 10.6, multiplied by 16, gives the solid Content of the *Cylinder* in Feet, and so of any other.

4. If the Circumference of a *Globe* or *Sphere* be given, the Area against it multiplied by 4, gives the superficial Content of such *Sphere*. Thus if the Circumference of a *Sphere* be 3 Foot, the Area against 3 is .7161972439 ; this multiplied by 4 gives 2.8647889756 the superficial Content of *Sphere*, whose Circumference is 3 Foot. Or thus double the Circumference given, and find the Sum in the first Column, the Area against it is the superficial Content sought ; thus twice 3 is 6, which found in the first Column, the Area against it is 2.8647889756 as above. And indeed, any Area in this Table is equal to the superficial Content of a *Sphere*, whose Circumference is half the Circumference against such Area respectively.

5. If the Circumference of a *Sphere* be found in the first Column, you have the Diameter in the third Column. Now if the Area between them be multiplied by two thirds of the Diameter, you have the Solidity of such *Sphere*.

S E C T. XVII.

The Use of the Instrument in Measuring of Timber and Stone.

Of Round Timber.

Problem I.

The Circumference and Length of any Round Tree, being given (in Foot-measure) to find the Content.

R U L E.

As 3.545 upon D, is to the Length upon C;
So is the Circumference upon D, to the Content upon C.

Example. Suppose the Length of a Tree be 20 Foot, and the Circumference 4 Foot, What is the Content?

Set 3.545, upon D, to 20 (the Length) upon C, then against 4 the Circumference upon D, you have 25.46 upon C, that is 25 Feet and almost an half, and so much is the Content.

Note, 3.545, is the Circumference of a Circle whose Area is 1.

Problem II.

The Circumference of a Round Tree being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Rule given in the last Problem will serve here, if instead of 3.545, you make use of 42.54. (which is found by multiplying 3.545 by 12.)

Example. The Circumference is 36 Inches, and the Length 10 Feet, What is the Content?

Set 42.54, upon D, to 10 upon C, and again 36 upon D, is 7.16 upon C, which is the Content sought.

Problem III.

The Circumference of any Round Tree being given, to find how much in Length it will take to make a Solid Foot.

First in Foot-Measure, the Proportion is,
As the Circumference upon D, is to 1 upon C,
So is 3.545 upon D, to the Length sought upon C.

Example. If the Circumference of a Tree be 36 Feet, How many Feet in Length will it take to make a Solid Foot?

Set

SECT. XVII. Of Round Timber. 251

Set 3 upon D, to 1 upon C, and against 3.45 upon D, is 1.396 upon C, that is, 1 Foot and .396 Parts. Now, so many times as this is contain'd in the Length of the Tree, so many Feet does the Tree contain.

Secondly, In Inch-measure, if the Circumference be given in Inches, *Say*,

*As the Circumference upon D, is 10 12 upon C;
So is 42.54 upon D, to the Length of Inches (upon C) to make a Foot.*

Example. The Circumference of a Tree is 48 Inches, How many Inches in Length will make a Foot?

Set 48 upon D, to 12 upon C, and against 42.54 upon D, is 9.42 upon C, that is 9 Inches and .42 Parts, the Length required.

Now supposing this Tree were 20 Foot long, the Content will be found to be 25.47, that is, 25 Foot, and almost a half, for 20 Foot is 240 Inches, and 9.42 Inches are contain'd 25.47, times in 240.

Note, The common way used for the Measuring of Round Timber is to girt the Tree about in the middle with a Line, and to take a fourth part of the Circumference for the Side of a Square equal to the Circles; but this is false, though most Men use it, for according to this Supposition it will take 12 Inches in Length of the Tree above-mention'd to make a Foot, and so the whole Tree would contain but 20 Foot, whereas the true Content is 25.47 Foot, as was shewn above.

Note, In all that hath been said of Round Timber, we suppose the Tree to be equally thick from end to end. But seeing most Trees are less towards

wards the Top than at the Root, the Circumference must be taken in the middle, and if the Ends differ much, it will be the more exact, if you divide the Length into two equal parts, and take the Circumference in the middle of each, and so find the Content as if they were two Trees. These Contents added together give the Content of the whole.

For Instance, There is a Tree of 12 Foot long, the Circumference at one end is 6 Foot, and at the other end 3 Foot, the Sum of these is 9, the half is 4.5, the Circumference in the middle, by which (and the Length) the Content will be found (by Problem I. to be 19.34 Foot.

But if this Tree be divided into two parts, as each may be 6 Foot long, the Circumference in the middle of the greater will be 5.25, by which the Content will be found to be 13.16 Foot.

And the Circumference in the middle of the lesser piece will be 3.75, and the Content of this will be 6.72, which added to the Content of the other, viz. 13.16, gives 19.88 Foot, the Content of the whole Tree: Whereas the Content found by one Circumference taken in the middle of the whole Tree, is but 19.34.

But in most Cases it will be sufficient to take the Circumference in the middle of the Tree, and so find the Content as in *Problem I.*

Observe also, in Taper-Trees, when it is said so much in length will make a Foot, it must be understood in that place only where the Circumference was taken, for it will not take so much in Length of the greater end of a Tree to make a Foot, as it will of the lesser.

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Yet, if by the Circumference taken in the middle you find the Length of a Foot, that Length so found will shew the whole Content of the Tree, as in Problem III.

Problem IV.

The Circumference of any Round Tree being given, either in Feet or in Inches, to find how many Solid Feet are contained in one Foot of the Length.

IF the Circumference be given in Feet, set 3.545 upon D, to 1 upon C; then against the Circumference upon D, is the Content upon C.

Example. Let the Circumference of a Round Tree be 4 Foot, How many Solid Feet are contained in one Foot of the Length of this Tree?
Answer, 1.273:

*For, As 3.545 upon D, is to 1 upon C;
So is 4 upon D, to 1.273 upon C.*

If the Circumference be given in Inches, make use of 42.54, instead of 3.545, and then the Proportion will hold as above.

of

Of Square Timber.

Problem V.

The Length and Side of any Piece of Square Timber, being given in Foot-Measure, to find the Content.

The Proportion is,

*AS 1 upon D, is to the Length upon C;
So is the Side upon D, to the Content upon C.*

Example. There is a Piece of Timber exactly Square, each side at either end being 1.5 Foot, and the Length 20 Foot, How many solid Feet doth this Piece contain?

Set 1 upon D, to 20 upon C, and against 1.5 upon D, is 45 upon C, which is the Content sought.

Problem VI.

The Side of the Square solid being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Proportion in the last Problem will hold here, if instead of 1 you make use of 12 for the first Term.

Example.

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Example. Let the Side be 18 Inches, and the Length 20 Foot, What is the Content in Feet?
Answer 45 : For,

As 12 upon D, is to 20 the Length upon C;
So is 18 the Side upon D, to 45 the Content upon C.

When the Ends are not exactly square (which doth often happen) take the Breadth and Thickness at the end in Feet and Parts, then multiply the Breadth by the Thickness, and this Product by the Length, the last Product will be the Content.

If the Breadth and thickness be given in Inches, and the Length in Feet, then by the Lines A and B on the Rule,

The Proportion is,

(1.) As 1 upon B, to the Breadth upon A;
So is the Thickness upon B, to a fourth Number,
viz. (the Area of the end in square Inches.)

(2) As 144 upon B, to the fourth Number upon A;
So is the Length upon B, to the Content sought.

But if any such Solid be greater at one end than at the other, the exact way to find the Content, is to find the Area of each Base (in Foot-measure) then find a Geometrical Mean betwixt the said Area's, then if the Sum of these three be multiplied by one third part of the Length, the Product is the Content.

SECT.

S E C T. XVIII.

The Use of the Rule in the Mensuration of Brick-work.

Brick-work is commonly measured by the Rod-Square, which is 16.5 Feet in Length, and as much in Breadth, and by consequence each square Rod doth contain 272.25 square Feet, for 16.5 multiplied by 16.5 is 272.25.

Note also, all Brick-work must be reduced to the Standard-measure of one Brick and a half thick.

Problem I.

The Length and Height of any Brick-Wall being given in Feet, to find the Content in square Rods at any thickness.

The R U L E.

Multiply the Height by the Length, and divide the Product by 272.25, the Quotient will be the true Content, if the Wall be just one Brick and a half thick :

If the thickness be more or less : Then say,

As 3, is to the Number of half Bricks in the Thickness of the Wall ;

So is the Quotient above found, to the true Content sought.

Or

Sect. XVIII. *Of Brick-work.* 257

Or having the Length and Height in Feet, you may find the Content more readily on the Rule by this Proportion.

*As 272.25 upon B, is to the Height upon A ;
So is the Length upon B to a fourth Number upon A, which is the true Content, if the Wall be one Brick and a half thick : If the thickness be more or less, say,*

*As 3 upon B, is to the half Bricks thick upon A ;
So is the fourth Number above found upon B, to the Content required.*

Example. There is a Wall 16 Foot high, and 97 Foot long : How many square Rod doth this Wall contain at one Brick and a half thick ? Answer 5 Rod and 7 tenth parts : For,

*As 272.25 upon B, is to 16 the Height upon A ;
So is 97 the Length upon B, to 5.7 upon A.*

Now if the Wall above-mention'd were two Bricks and a half thick, it would contain 9 Rod and an half : For,

*As 3 upon B, is to 5 (the half Bricks in two and a half) upon A ;
So is 5.7 the Content at one Brick and a half thick upon B, to 9.5 the Content sought upon A.*

But if a Wall of 16 Foot high, and 97 Foot long were but 1 Brick thick, the Content would be but 3 Rod and 8 tenth parts : For,

As

As 3 upon B, is to 2 (the half Bricks in one) upon A;

So is 5.7 (the Content at one and a half thick) upon B. to 3.8 the Content upon A.

Now, that (in all these Cases) you may find the Content at one Operation: I shall here lay down certain fix'd Numbers for any Thickness hereafter express'd,

viz. at	$\left. \begin{array}{l} 1 \\ 1 \frac{1}{4} \\ 2 \\ 2 \frac{1}{2} \\ 3 \\ 3 \frac{1}{2} \\ 4 \\ 4 \frac{1}{2} \\ 5 \end{array} \right\}$	Bricks thick.	408.37
			272.25
			204.19
			163.35
			136.12
			116.68
			102.10
			90.75
			81.75

By these Numbers the Content of any Wall, any Thickness here mentioned may be found either by the Pen or Rule, and the Proportion is,

As the Number proper to the Thickness, is to the Height;

So is the Length to the Content.

Example. There is a Wall four Bricks and a half thick, 7 Foot high, and 85 Foot long: How many square Rod doth this Wall contain?

The proper Number for four Bricks and a half thick, is 90.75.

Therefore

Therefore, Set 90.75 upon A, to 7 the Height upon B; then against 84 the Length upon A, is 6.555 upon B; that is, six Rod and a half, and something more.

Note, The Decimal parts of a Rod may be reduced into Feet by this Proportion.

As 1.00 upon A. is to 272.25 upon B;

So is any Decimal upon A, to the Number of Feet upon B.

And thus you may find the Decimal in the last Example, viz. .555 is equal to 151 Feet.

Also, $\left\{ \begin{array}{l} \frac{1}{4} \text{ or } .25 \\ \frac{1}{2} \text{ or } .50 \\ \frac{3}{4} \text{ or } .75 \end{array} \right\}$ of a Rod $\left\{ \begin{array}{l} 66 \\ 132 \\ 198 \end{array} \right\}$ Feet.

Problem II.

The Height of any Wall being given in Feet, to find how many Feet in Length it will take to make a Square Rod at any Thickness above-mention'd.

R U L E.

SET the Height upon A, to the Number proper for the Thickness upon B, then against 1 upon A, is the Length sought upon B.

Example

Example. There is a Wall one Brick and thick, and 8 Foot high: How many Feet in Length will it take to make a square Rod? Answer, For, The proper Number for one and a half thick 272.25. Therefore,

As 8 (the Height) upon A, is to 272.25 upon
So is 1 upon A, to 34 upon B;

S E C T. XIX.

The Use of the Rule in the Mensuration of Superficies, as Board, Paving, Cieling, Wall scotting, Flooring, Tyling, &c.

Note, In all Square or Oblong Superficies the Area or Content may be found by this Proportion :

As 1 upon A, to the Breadth upon B ;
So is the Length upon A, to the Content upon B.

This Proportion will hold in Triangular Superficies also, if you take half the Perpendicular for the Breadth, and the Side upon which it falls for the Length : And in all Cases, if the Sides be given

in $\left\{ \begin{array}{l} \text{Inches} \\ \text{Feet} \\ \text{Yards} \end{array} \right\}$ the Area will be $\left\{ \begin{array}{l} \text{Inches,} \\ \text{Feet, or} \\ \text{Yards.} \end{array} \right\}$ Square

XIX. *Of Board Measure.* 261

but in many Cases the Sides are given in one kind Measure, and the Area required in another : shall therefore give some Examples of each.

I. *For Board.*

the Length and Breadth be both given in Inches, and the Content sought in Feet. *Say,*

144, is to the Breadth;
as is the Length, so the Content.

Example. A Board is 14 Inches broad, and 156 Inches long: How many square Feet doth it contain? *Answer* 15.16. *For,*

144 upon A, is to 14 the Breadth upon B;
as 156 the Length upon A, so 15.16 the Content upon B;

the Breadth be given in Inches, and the Length in Feet: *say,*

12, is to the Breadth;
as is the Length, so the Content.

Example. A Board is 14 Inches broad, and 13 Feet long: What is the Content in Feet? *Answer* 15.16. *For,*

12 upon B, is to 14 the Breadth upon A;
as 13 the Length upon B, so 15.16 the Content upon A.

II. *Of*

Of Paving.

Pavier's Work is measured by the Yard Square, which contains 9 square Feet: The most natural way is to measure the Sides with a Yard divided into 100 equal parts, and then multiply the Breadth by the Length, the Product will be the Content in square Yards: But if the Sides be given in Feet, say,

As 9 is to the Breadth;
So is the Length, to the Content.

Example. There is a Court pay'd with Stone the Breadth is 64 Feet, and the Length 95 Feet: What is the Content in square Yards? Answer 67.55. For,

As 9 upon A, is to 64 the Breadth upon B
So is 95 the Length upon A, to 67.55 the Content upon B.

III. *Of Cieling.*

Cieling and also Wainscoting, are measured by the Yard Square, and therefore what is said in the Example above, may be understood here.

IV. *Of*

IV. Of Flooring and Tyling.

These are both measured by the *Square*, which contains 100 square Foot, the Side being 10: Therefore if the Length and Breadth be given in Feet, multiply one by the other, and cut off Figures in the Product, the Remainder shews Content in Squares, and the Figures cut off are parts of a Square, or odd Feet.

Or by the Rule, say,

As 100, is to the Breadth in Feet;
So is the Length in Feet, to the Content in Squares.

Example. There is a Floor 66 Foot broad, and 75 Foot long. How many Squares doth it contain? Answer, 49.5. For, as 100 upon A, is to 66 the Breadth upon B; so is 75 the Length upon A, to 49.5 the Content upon B.

The like for Tyling.

Lastly, The Breadth of any Superficies being given in Feet or Yards, to find how many in Length (either) it will take to make a Foot or Yard square.

The Proportion is,

As the Breadth is to 1;
So is 1, to the Length sought.

Example.

Example. A Board is 1.5 Foot broad: How much Length will it take to make a Foot square? Answer, .667. For,

As 1.5 (the Breadth upon) A, is to 1 upon B;
So is 1 upon A, to .667 upon B.

So that if one side of a piece of Board be 1.5 Foot, the other side must be .667 of a Foot, to make a Square-Foot; for .667 multiplied by 1.5 is 1.

In Inch-measure the Proportion is,

As the Breadth in Inches, is to 12;

So is 12 to the Length in Inches to make a Foot.

And, As the Breadth in Inches is to 36,

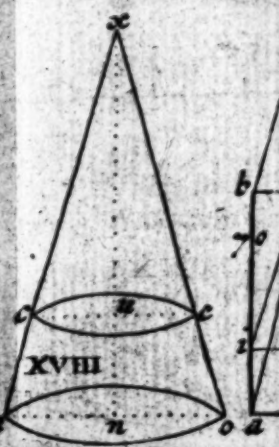
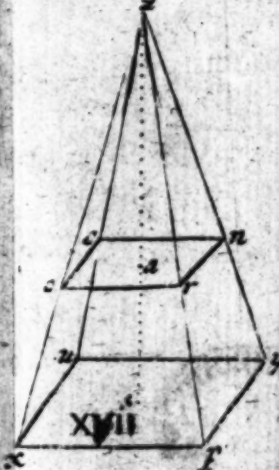
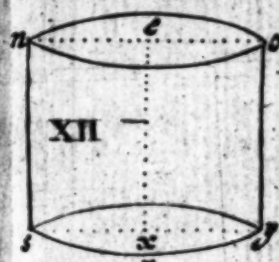
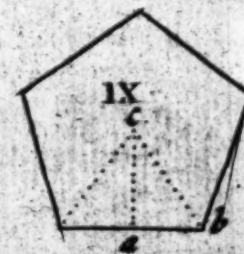
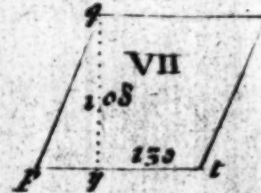
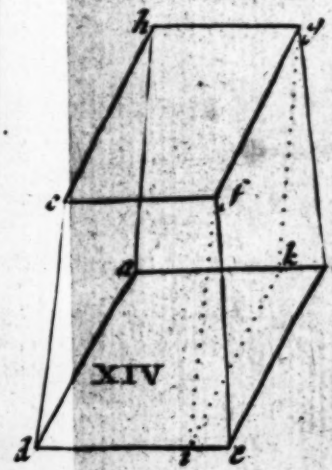
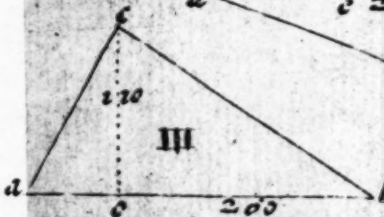
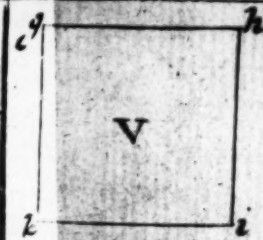
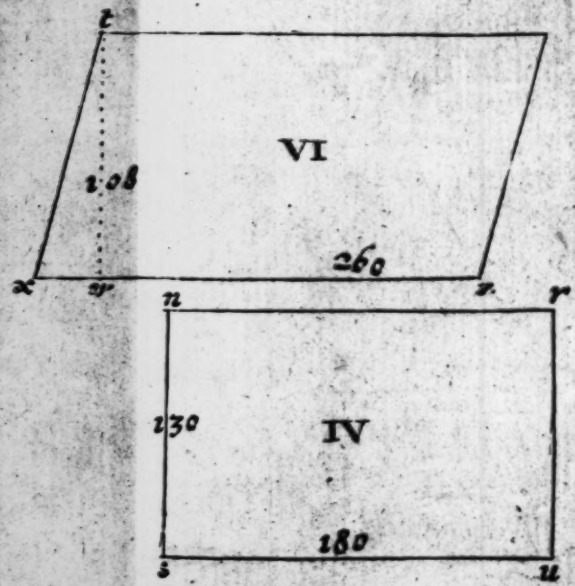
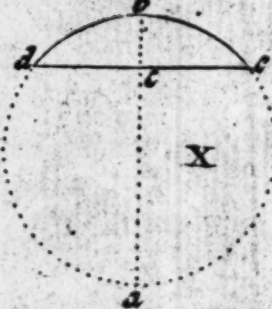
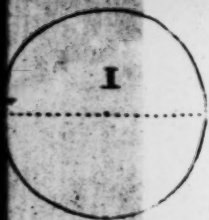
So is 36

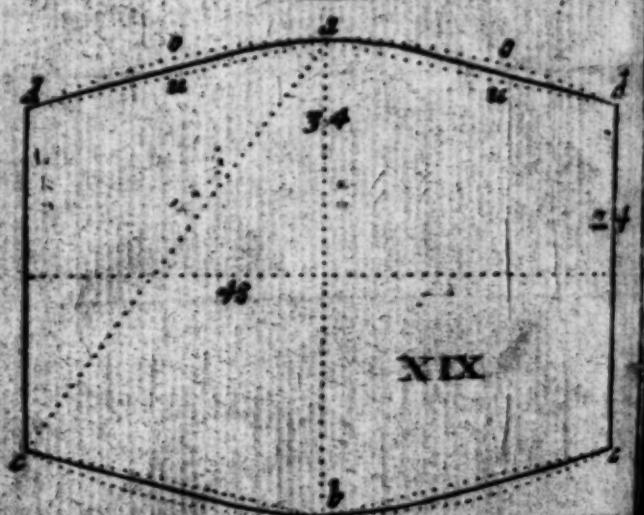
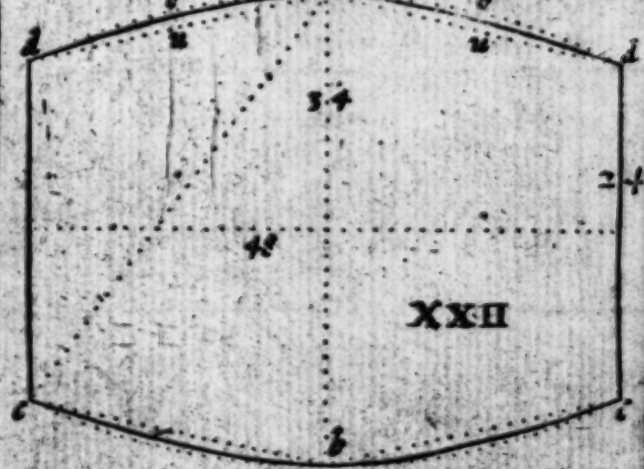
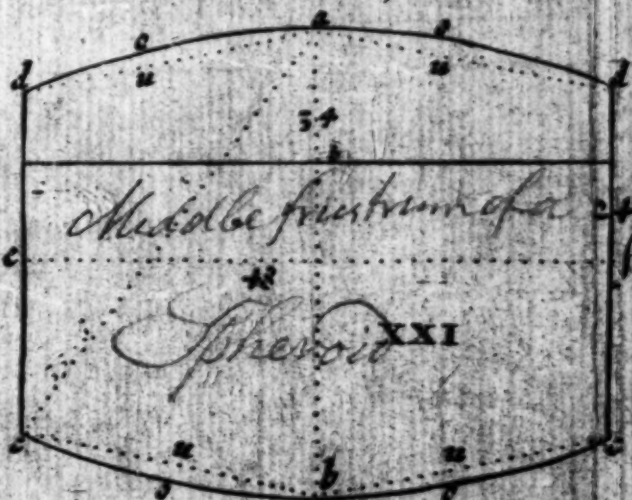
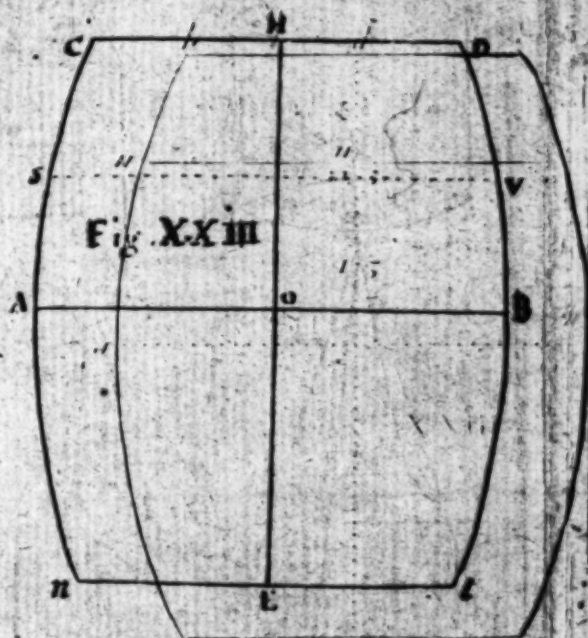
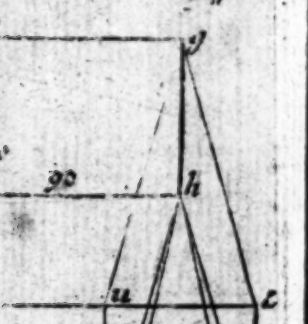
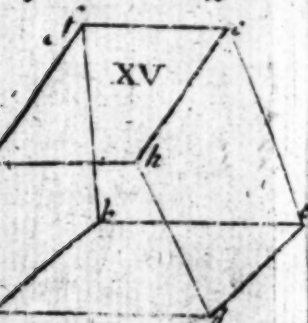
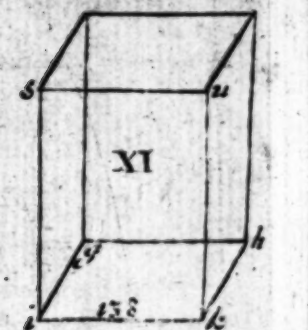
To the Length in Inches to make a Yard.

Example. A Board is 18 Inches broad, how many Inches in Length will make a Foot?

Set 18 (the Breadth) upon A, to 12 upon B, then against 12 upon A, is 8 upon B; and so many Inches in Length will make a Foot of this board; for 18 multiplied by 8, gives 144 the Square-Inches in a Square-Foot.

A P P E N D I X





Place this before the Appendix.

APPENDIX.

Of Conical Sections.

IN the Doctrine of Conicks there are these two things (very different) to be separately considered. *First*, What Figures or Curve-Lines arise from the Section of a Cone by a Plain. And *Secondly*, What is the nature of such Figures or Curve-Lines, considered with respect had to such supposed original, by cutting a Cone.

See Dr. Wallis
de Con. Sect. and
Treatise of Algebra, pag. 262.

DEFINITION.

1. A Cone is a Solid described, when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the place from whence it first moved : The Axis of the Cone is that fixed Side about which the Triangle is moved : The Base of the Cone is a Circle, which is described by the containing Side that is moved about.

N

Of

Of the several Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis from the Vertex to the Center of the Base the Section will be a *Triangle* as SVB. Fig. 1 and 2.

2. If a Cone be cut by a Plain at Right Angle to its Axis, (or if a Right Cone, Parallel to its Base) the Section will be a *Circle*, whose Diameter let be S d B, Fig. 1.

3. If a Cone be cut by a Plain Parallel to the side of the Triangle (SV) the Crooked Section C c N c c C, will be a *Parabola*; in which P N, is the Diameter, or Axis, and d c an ordinately applied Line. Vid. Fig. 1.

4. If a Cone be cut by a Plain that will also cut one of the sides of the Triangle produced in E (Fig. 1.) the Crooked Section O o o A o o O, is a *Hyperbola*, in which A H is the intercepted Diameter: A E the Transverse Diameter; and b o, an ordinately applied Line.

5. If a Cone be cut by a Plain through both the sides of the Triangle under the Vertex V, the Crooked Section, a o o A o o will be an *Ellipsis*, in which a A is the Diameter, and D o, and ordinately applied Line. Vid. Fig. 2.

There are all the Sections that can be made by cutting a Cone by a Plain. I shall treat only of the four last, and of these but so much as I conceive necessary to my present Design, which

only to explain the Nature and Original of some Solids which are frequently mentioned in Books and do sometime occur in the Practice of Gauging.

Of a Parabola.

IN the *Parabola* (Fig. 1.) Let the Cone be cut by a Plain (S d B) parallel to the Circular Base (S P B) it will appear,

1. That S d is equal to S P, for N P is parallel to S V, and S d B, is parallel to S P B.

2. That S P multiplied by P B is equal to P C squared. Therefore $S d \times d B = d c q.$

3. As N P, is to P C squared;
So is N d, to d c squared. Therefore,

4. As N P, is to N d;
So is P C squared, to d c squared, which is the general Property of a *Parabola*.

Problem I. Figure 3.

To describe a Parabola in Plano.

DRaw the straight Line O P O, and upon the middle Point P, erect the Perpendicular A P, compleat the Triangle by drawing the Lines A O, A O, then through A P (the Axis) and parallel to O P O (the Base) draw a competent number (the more the better) of straight
N 2 Lines

Lines SDS. Now wheresoever D be taken, it will be

$$AP : PO :: AD : De. \text{ And } PO \times De = D o q.$$

This gives the Points o. o. (as many as you please on both sides) through which the Curve Line Ooo Aooo being drawn, you have the *Parabola* required.

Or thus,

Having drawn the *Parallelogram* ONVO, (Fig. 3.) divide AN into any number of equal parts, and suppose AP divided into as many equal parts as the Square of the number of parts in AN, as here AN is divided into 4 equal parts, and AP is supposed to be divided into 16 equal Parts. Let Lines be drawn from N. n. n. n. parallel to AP. Now, if from a Scale of equal Parts, of which 16 are equal to AP, you make the first $nc = 1$. the second $nc = 4$. the third $nc = 9$. and the fourth $NO = 16$. the Points C. C. C. O. are in the *Parabola*, and by a competent number of Points thus found, the *Parabola* may be drawn.

Note, 1. The *Parabola* OcAO, is two-thirds of ONVO, the *Parallelogram* circumscribed, and the Triangle OAO, is three fourths of the *Parabola*; therefore if the *Parallelogram* be 3, the *Parabola* is 2 and the *Triangle* 1.

2. If the *Parallelogram* PANO, be turned about the Line AP, it will generate the *Cylinder* ONVO.

ONVO, and by the same conversion there will be generated by the Curve-Line O c c c. the *Parabolick Conoid*, O C A O O, and by the streight Line O A, the Cone O A O, and these three Solids are as 3. 1 and a half, and 1; for the *Parabolick-Conoid*, is half the circumscribing *Cylinder*, and the *Cone*, one third part thereof. Now, all these properties being common to every *Parabola* (as is demonstrated by Dr. Wallis in his *Mechanica*, Part 2 Chap. 5.) the Content of any *Parabolick Conoid* is easily found, as shall be shewn in the next Problem.

Problem II.

There is a *Parabolick Conoid* (Fig. 3.) O A O, the Diameter of the Base is 32 Inches, and P A, the Altitude 45.7143 Inches : What is the Content in Ale-Gallons ?

Multiply the Area of the Base by half the Altitude, the Product gives the Content sought.

Example. The Diameter is 32. The Area is 2.8519, this multiplied by half the Altitude (*viz.*) 22.8571 gives 65.18 the Content sought.

Or, Multiply the Square of the Diameter by half the Altitude, and that Product by .0027851, the last Product will be the Content in Gallons.

Problem III.

There is a Frustum of a Parabolick Conoid, (Fig. 3.) whose Bases are Parallel; O P O, the Diameter of the greater Base is 32 Inches: C D O, the Diameter of the lesser Base 24 Inches, and P D the Altitude 20 Inches: What is the Content in Ale-Gallons?

FROM the square of P O (the Semidiameter of the greater Base) subtract the square D O (the Semidiameter of the lesser Base) then say,

*As that Difference,
Is to P D the Altitude of the Frustum;
So is the Square of P O,
To P A the Axis of the whole Conoid.*

In this Example it will be

$$P o q. - D o q: P D :: P o q: P A$$

$$112 : 20 :: 256 : 45.7143$$

Now, if from P A you subtract P D, you have the Altitude of the *Parabolick Conoid*, whose Base is C D O. And since every *Parabolick Conoid* is equal to half a *Cylinder* of the same Base and Altitude, the Area of the Base O P O multiplied by half the Altitude P A, gives the Content of the *Parabolick Conoid*, whose Base is O P O, and Altitude A P. And the Area of the Base C D O, multiplied by half the

the Altitude $D A$, gives the Content of the *Parabolical Conoid* whose Base is $C D o$, and Altitude $D A$. Therefore, if from the Content of the former you subtract the Content of the latter, you have the Content of the Frustrum $C D o, O P o$.

Example. $C D o$ is 24 Inches, the Area is 1.6042, this multiplied by half $D A$ (*viz.*) 12.8571, gives 20.62 Gallons, the Content of the *Parabolical Conoid* $O C C A O O$, this subtracted from 65.18 (the Content of the *Parabolical Conoid* $O C A o o$) leaves 44.56 Gallons, the Content of the Frustrum ($C D o, O P o$), which was demanded. But the Content of any such Frustrum may be more readily found, thus,

To the Area of the greater Base, add the Area of the lesser Base, the Sum multiplied by half the Altitude, gives the Content.

Example. In the Frustrum last mentioned.

The Diameter $O P o$, being 32 } 2.8519
Inches, the Area is _____

The Diameter $C D o$, being 24 } 1.6042
Inches, the Area is _____

Sum is _____ 4.4561

This multiplied by half the Altitude (*viz.* 10.) gives 44.561 Gallons the Content, as by the last.

Now, Suppose the Fruſtum $O B G O$, be equal to the Fruſtum $O C D O$, and the Baſe $O P O$, common to both, this is what ſeveral Writers would have to repreſent a common Caſk, *viz.* a *Butt*, *Hogſhead*, *Barrel*, &c. But it is plain from the nature and conſtruction of the Figure, that no ſuch Caſk can be in the Form of theſe two Fruſtums abutting upon the common Baſe $O P O$. But if it were practicable to make a Caſk in this Form, if the Diameters, and the Length be as above-mentioned, the Content will be 89.122 Gallons.

To find the Content upon every Inch of the Fruſtum. $C D O$. $O P O$.

From the Square of ($O P O$) the Diameter of the greater Baſe, ſubtract the Square of ($C D O$) the Diameter of the leſſer Baſe, and divide this difference by $D P$ the Depth, the Quotient is the common Difference of the Squares of the ſeveral Diameters, taken one in the middle of every Inch of $D P$. But for the Diameter in the middle of the firſt Inch next the leſſer Baſe, take half this common Difference, and add it to the Square of $C D O$, the Sum is the Square of the Diameter in the middle of that Inch, to this add the common Difference, the Sum will be the Square of the Diameter in the middle of the ſecond Inch, and ſo of the reſt. But having got the Squares of the Diameters in the middle of the firſt, ſecond, and third Inch, find the Area's of thoſe Circles in Gallons, the Difference of theſe Areas will be equal.

Example.

Example.

$$O P O = 32 \text{ Square} = 1024$$

$$C D O = 24 \text{ Square} = 576$$

$$\text{Difference} = 448$$

448 divided by D P 20, is 22.4 the common Difference.

$$\text{Square of } C D O = 576$$

$$\text{half com. differ.} = 11.2$$

$$\text{Square of the Diameter in the middle of the first Inch } \} = 587.2$$

$$\text{Common Difference} = 22.4$$

$$\text{Square of the Diameter in the middle of the second Inch } \} = 609.6$$

$$\text{Common Difference} = 22.4$$

$$\text{Square of the third Diameter} = 632.0$$

The Squares of these Diameters thus found, multiplied severally by .002785, give the Area or Content of the first, second, and third Inch.

		Areas.	Diff.
Viz. {	1	1.6354	.0624
	2	1.6978	
	3	1.7602	

And so of the rest, as in the following Table ; in the last Column of which are the common Difference of the Areas. In the third Column, you have the Area or Content of every Inch. The first

first Number in the second Column, *viz.* 44.5640 is the Sum of all the Area's in the third Column, that is, the Content of the whole Tun: From which the several Area's continually subtracted, leaves the number of Gallons contained in the Tun at any Depth. Thus against 10 Inches deep, you have 25.402 Gall. in the Tun, and so for any other Depth.

Inches of the Tun's depth.	Content at any depth.	Area's.	Com. Diffe.
20	44.5640	1.6354	.0624
19	42.9286	1.6978	.0624
18	41.2308	1.7602	.0624
17	39.4706	1.8226	.0624
16	37.6480	1.8850	.0624
15	35.7630	1.9474	.0624
14	33.8156	2.0098	.0624
13	31.8058	2.0722	.0624
12	29.7336	2.1346	.0624
11	27.5990	2.1970	.0624
10	25.4020	2.2594	.0624
9	23.1426	2.3218	.0624
8	20.8208	2.3842	.0624
7	18.4366	2.4466	.0624
6	15.9900	2.5090	.0624
5	13.4810	2.5714	.0624
4	10.9096	2.6338	.0624
3	8.2758	2.6962	.0624
2	5.5796	2.7586	.0624
1	2.8210	2.8210	.0624
Content = 44.5640			

of

Of a Parabolick Spindle.

IN Fig. 5. RH is 40, $RS = bE$ 20. AR 16,
 Rb 12, and bH 4. And

$$AR : RHq :: Ah : bEq.$$

$$16 : 1600 :: 4 : 400.$$

Therefore (the other side being made like to this) the Curve Line $GCAEH$ is a *Parabola*. Now if this *Parabola* be turned about upon the Line GH , it will generate the Solid $GAHBG$, which is called a *Parabolick Spindle*. And every *Parabolick Spindle* is to its circumscribing Cylinder, as 8 to 15. (*Vid.* Dr. Wallis's *Mechanica*, Part II. Chap. 5. Prob. 12.)

Problem IV. Figure 5.

To describe a *Parabolick Spindle*, the middle *Frustum* whereof shall Represent a Cask whose Diameter at the Bung AB , is 32 Inches, the Diameters at the Heads CD and EF are each 24 Inches, and the Length PS 40 Inches. And to find the Content of the middle *Frustum* thereof, representing the Cask proposed.

1. DRAW the straight Line zz , and at right Angles to it, draw the Lines AB , CD , and EF , so as RP and RS , be each = 20. Make

Make $AR = 16$. CP , bR and Es , each $= 12$, so shall Ab be $= 4$. And because it is a *Parabola*, it will be

$$Ab : AR :: bEq : RHq \\ 4 : 16 :: 400 : 1600$$

The Square-Root of 1600, is 40, $= RH$ or RG .

2. To find a Diameter in the middle of every Inch of RG , or RH . Through the Point A , draw IK Parallel, and equal to the Line $H-G$, let IA and GR , be each divided into half Inches, which here will be 80, the Square of this is 6400, by which divide AR 16, the Quotient will be .0025, this multiplied by the Squares of all the odd Numbers, viz. 1, 3, 5, 7, &c. to 79, the several Products give the Length of the Lines ne , which taken from the Line AR , leaves the Lines eo , the Semidiameters of the *Parabolick Spindle*, (one in the middle of every Inch of GR .) But to contract this Work, we need but the three first Diameters, the rest are obtained by subtracting the common Difference; as will appear by the following Operation.

$$\begin{array}{l} .0025 \\ \text{Multipli-} \\ \text{ed by} \end{array} \left\{ \begin{array}{l} 1 \\ 9 \\ 25 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} .0025 \\ .0225 \\ .0625 \end{array} \right\} \text{ each of}$$

these taken from AR 16, the three Remainders will

$$\begin{array}{l} \text{be} \end{array} \left\{ \begin{array}{l} 15.9975 \\ 15.9775 \\ 15.9375 \end{array} \right\} \begin{array}{l} \text{these mul-} \\ \text{tiplied by 2} \\ \text{give} \end{array} \left\{ \begin{array}{l} 31.995 \\ 31.955 \\ 31.875 \end{array} \right\} \begin{array}{l} \text{the} \\ \\ \end{array} \text{three}$$

three Diameters, one in the middle of every Inch next to A B; to find the common Difference, take the second Diameter from the first, there rests .040; take the third from the second, the Remainder is .080, take .040 from .080. there rests .040, the second Difference, which is the same throughout; and by this common Difference the rest of the Diameters are thus found. Take the first Difference (*viz.* .040) from the first Diameter, the Remainder is the second Diameter; add the second Difference to the first Difference, and subtract their Sum from the second Diameter, the Remainder will be the third Diameter; and so of the rest, as in the following Table. In the second Column of which you have the second Difference (which is the same throughout) in the third Column are the first Differences, in the fourth Column you have the true Diameters of the Spindle, one in the middle of every Inch of R P. and in the last Column are the Areas to these Diameters, that is, the Content of so many circular Plains of one Inch thick, which added together make 48.249 Ale-Gallons, the Content of the Part B D C A, this doubled is 96.498, the Content of the middle Frustum C A E F B D, representing the Cask proposed.

Note, The Areas in this Table were found by multiplying the Squares of each Diameter by .002785. But this Work may be much shorten'd; for if the Squares of the several Diameters in the fourth Column be added together, and their Sum multiplied by .002785, the Product will be 48.349 = the Sum of the Areas in the last Column.

A

A Table shewing the Content of half the middle Frustrum of the *Parabolick Spindle*, CAEFBD. Fig. 5.

	Second Differ.	First Differ.	Diameter in the middle of every Inch of R P.	Area in Gallons, or the Content of every Inch from R to P.
1			31.995	2.851
2	40	40	31.955	2.844
3	40	80	31.875	2.830
4	40	120	31.755	2.808
5	40	160	31.595	2.780
6	40	200	31.395	2.745
7	40	240	31.155	2.703
8	40	280	30.875	2.655
9	40	320	30.555	2.600
10	40	360	30.195	2.539
11	40	400	29.795	2.473
12	40	440	29.355	2.400
13	40	480	28.875	2.322
14	40	520	28.355	2.239
15	40	560	27.795	2.152
16	40	600	27.195	2.060
17	40	640	26.555	1.964
18	40	680	25.875	1.865
19	40	720	25.155	1.762
20	40	760	24.395	1.657
			Sum	48.249
			Content of the Cask	96.498

Note, If the part A B D C were a Tun standing upon its lesser Base D C, and from the whole Content thereof, viz. 48.249, you subtract the first Area, viz. 2.851, the Remainder 45.398, will be the Content at 19 Inches deep ; and if from this you take the second Area, you will have 42.554, the Content at 18 Inches deep, and proceeding with the rest in the same manner, you will have a Table shewing the Content at any Depth.

Of an Hyperbola.

I N Fig. 1. the Cone S V B, is cut by a Plain H A, which also cuts one Side of the Triangle produced in E, therefore the crooked Section O o O A o o O, is an *Hyperbola*. In which E A is the transverse Diameter, and A H the intercepted Diameter. Now if the Cone be cut by another plain Parallel to the Base, this Section will be a Circle, whose Diameter is S H B : And $SH \times HB = HOq$, therefore H O is an ordinate applied Line, and if H O be a Line ordinate applied, it will be

$$AH \times EH : Ab \times Eb :: HOq : boq :$$

Problem

Problem V. Fig. 4.

To find the Content of an Hyperbolick Conoid, or the Content of any part thereof in Ale-Gallons.

IF the *Semi-Hyperbola* A S S S D, be turned round about upon the Line A D, it will describe the *Hyperbolick Conoid*, S S A o O. Now, if we put the Transverse Diameter = t , the Intercepted Diameter = d , the Magnitude of the *Hyperbolick Conoid* = A, and the Magnitude of a Cylinder of the same Base and Altitude = B, it will be

$$6t + 6d : 3t + 2d :: B : A.$$

That is, (1.) To 6 times the Transverse Diameter A E, add 6 times the intercepted Diameter A D, the Sum is your Divisor.

(3.) To 3 times the Transverse Diameter, add twice the Intercepted Diameter, by this Sum multiply the Content of the Cylinder N V O S, the Product is your Dividend; which divided by the Divisor above found, the Quotient will be the Content of the *Hyperbolick Conoid*, whose Base S D O, is the Base of the Cylinder N V O S, and Altitude A D equal to S N the Altitude of the Cylinder; and this will hold wheresoever the Base be taken: For suppose the first S d o, next the Vertex A, be taken for the Base of the *Hyperbolick Conoid*, S A o, (and of the Cylinder S n n o,) the Content of this *Hyperbolick Conoid* may be found as above, and so the Content of any Frustum, S D O o D S may be likewise found,

found, for if the little *Hyperbolick Conoid* S A o d, be taken from the *Hyperbolick Conoid* S S S w o o O, there remains the Frustum S D O o d s, and thus the Content of the whole, and also the Content of any Frustum, and consequently of every Inch of the *Hyperbolick Conoid* may be readily found, as will appear in the following Example.

Example. Fig. IV.

Let A E, the Transverse Diameter, and A D, the Intercepted Diameter, be each 32 Inches, and S D O the Diameter of the Base 45.255 Inches, to find the Content of the *Hyperbolick Conoid* in Ale-Gallons.

1. Six times A E is 192, and 6 times A D is 192, the Sum of these is 384, for the first Term of the Proportion.

2. Three times A E is 96, and twice A D is 64, the Sum of these is 160, for the second Term.

3. The Diameter of the Base S D O is 45.255 (equal the Square Root of 2048) and the Area of this Circle is 5.704 Gallons; this multiplied by 32 (equal the Altitude S N) gives 182.528 the Content of the Cylinder N S O V, and this is the third Term of the Proportion, which will stand thus ;

$$384 : 160 :: 182 : 528.$$

Now multiply the *third* Term by the *second*, and divide that Product by the *first*, the Quotient or *fourth* Term will be 76.053 Ale-Gallons, which is the Content of the *Hyperbolick Conoid* required.

And by the same Method the Content of the little *Hyperbolick Conoid*, whose Altitude d A, is 4 Inches, and the Diameter of its Base (S d o) 16.97 Inches,

Inches, will be found to be 1.545 Gallons. This taken from the Content of the *Hyperbolick Conoid* above-found, the Remainder is 74.508 Ale-Gallons, the Content of the Frustrum *S D O o d S*; and so for any other Frustrum.

Problem VI. Figure 6.

To describe an Hyperbola in Plano Geometrically.

Draw a straight Line *R R*, and from the middle Point *W*, let fall the Perpendicular *W B*, then divide *W R* in any number of Parts, and draw the Lines *r o G*, *r o H* parallel to *W B*, then set one Point of your Compasses in the Line *W B*, suppose at *A*, and extend the other to *r*, where keep it fast whilst the first Point, with the same Extent, is removed from *A*, to the Line *r G*, it will fall upon the Point *O*, and so on the other side. I say, the Points *o o o*, are in the *Hyperbola*, and by a competent Number of such Points thus found, the *Hyperbola* may be drawn. *W A* is half the Transverse Diameter, *A B* the intercepted Diameter, and *B o* an ordinately applied Line, (*Vid. Dr. Wallis's Mechanica*, Part II. Pag. 553.)

Note, If the *Hyperbola O o A o O*, be turned about upon the Line *z z*, it will describe a Solid. *O o A o o O F D E O*, which may be called a *Hyperbolick Spindle*.

Problem

Problem VII. Fig. 6.

To describe a Hyperbolick Spindle, the middle Frustum of which shall represent a Cask, whose Diameter at the Bung A D, is 32 Inches, the Diameters at the Heads, O E, and O F, are each 24 Inches, and the Length G H, 40 Inches, and to find the Content of this Frustum in Ale-Gallons.

Draw the right Line $z z$, and at Right Angles thereto, the Line $W D$, make $B G$ and $B H$, each equal to 20 (the Semi-length of the Cask proposed) through the Points G and H , draw the Lines $r E$, and $r F$ parallel to $W D$, make $G O$ and $H o$, each equal to 12, and $B A$ 16, then make $B b$ equal to $G o$, so shall $A b$ be 4. I say, the Square $b o$, (400) divided by $A b$, (4) is equal to 100, from which subtract 4, the Remainder (96) is the Transverse Diameter of this Hyperbola, make $A W$ equal to 48, (the half of 96) and through the Point W , draw the Line $R R$ parallel to $z z$, and let the Line $r o$ be parallel to $W D$; I say, the Square of $W A$, more the Square of $W r$, is equal to the Square of $r o$, of which the Square-Root is the Length of the Line $r o$. Now the Line $r o$, taken from the Line $W B$, the Remainder is the Line $o g$, which doubled gives $o e$, a Diameter in the Spindle; and this will hold wheresoever r be taken, and so innumerable Diameters $o e$, may be found on both Sides $A D$, to the ends of the Spindle. But for our present purpose, it will be sufficient to find

find a Diameter in the middle of every Inch of the part A D E O, as suppose the first Diameter o g e, be half an Inch from A D, the first W r is .5, the Square of this (*viz.* 25.) added to the Square of W A (2304) is equal to the Square of r o, (2304.25) the second W r, will be 1.5, the third 2.5, &c.) the Squares of which you have in the first Column of the following Table. In the second Column are the Sums of the Squares of W A and W r, that is, the Squares of r o. In the third Column are the Square-Roots of the Numbers in the second Column, that is, the Length of the Lines r o. Now each of these Numbers taken from W B 64, leave the Lines o g, which doubled are the Diameters o e, as in the fourth Column. In the last Column are the Areas of these Circles in Ale-Gallons, that is, the Content of so many Circular Plains of one Inch thick, the Sum of these is 49.1 (equal the part A D E O) which doubled is 98.2, equal to the middle Frustrum O E D F O A. The Content here found is not Mathematically True, but near enough for any ordinary Practice, and by the Method here laid down, the Content may be found so near the Truth, that the difference shall be less than any quantity assignable.

Now, Let the part D E O A, represent a Tun, and suppose it to stand upon the lesser Base E o. If from the whole Content thereof, *viz.* 48.1, you subtract the first Area, *viz.* 2.851, the Remainder 45.249, will be the Content at 19 Inches deep; and if from this you subtract the second Area, *viz.* 2.843, you will have 42.406, the Content at 18 Inches deep; and proceeding with the rest after the same manner, you will have a Table shewing the Content of this Tun at any Depth.

A TABLE shewing the Content of every Inch of half the middle Frustum of the Hyperbolick Spindle Eo Ao FD (Fig. VII.) in Ale Gallons.

Squares of W r.	Sum of the Squares of W A. and W r.	Square Roots of the Square of (r o) or the length of the Lines r o.	Diameters o c	Areas of the circles in Ale. Gallons.
.25	2304.25	48.003	31.995	2.851
2.25	2306.25	48.023	31.953	2.843
6.25	2310.25	48.065	31.879	2.829
12.25	2316.25	48.127	31.745	2.807
20.25	2324.25	48.210	31.579	2.777
30.25	2334.25	48.314	31.372	2.741
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.835	2.648
72.25	2376.25	48.747	30.506	2.592
90.25	2394.25	48.931	30.138	2.520
110.25	2414.25	49.135	29.730	2.462
132.25	2436.25	49.358	29.283	2.388
156.25	2460.25	49.601	28.798	2.310
182.25	2486.25	49.862	28.275	2.227
210.25	2514.25	50.142	27.715	2.139
240.25	2544.25	50.441	27.119	2.048
272.25	2576.25	50.757	26.486	1.954
306.25	2610.25	51.090	25.819	1.857
342.25	2646.25	51.442	25.117	1.753
380.25	2684.25	51.809	24.380	1.656
Sum is —————				48.100
This doubled is —————				96.200
The Content of the Frustum Eo Ao E D.				

of

Of an Ellipsis.

IF the Cone $S e B d V$ (Fig. 2.) be cut by a Plain ($A a$) through both the Sides of the Triangle, the crooked Section $a o o A o o$ will be an *Ellipsis*, in which $A a$, is the Transverse Diameter, or Axis, and $A D$, a Diameter intercepted.

Let the Cone be cut by another Plain ($S B$) parallel to the Base $S e B d$, it will be $S D \times B D = D o q$, and this will hold wheresoever D be taken, and give the Lines $D o$, which are called Ordinately applied Lines, and the Lines $o D o$ (cutting the Line $a A$, at Right Angles) are called Conjugate Diameters of the *Ellipsis*.

Problem VII. Fig. 7.

Given the Axis of the Ellipsis, and its greatest Conjugate Diameter (cutting the Axis at Right Angles) to describe the Ellipsis.

DR A W a straight Line, and make $A C$, equal to the Axis, given, through the middle Point B , and at Right Angles to $A C$, draw the Line $F L$, and from the Point B , set off half the Conjugate Diameter to O and G , draw the Lines $D H$, $d h$, parallel to $F L$. Join $B D$, and $B d$, upon the Center B , and with the interval $B A$, describe the Circle $F A L C$. I say, $B D q - B E q = E D q$. And $B F : B o :: E D : E O$. This will hold wheresoever E be taken, and give the Points

Points o o, (as many as you please on both Sides) through which the *Ellipsis* may be drawn.

Of a *Spheroid*. Fig. VII.

IF the *Semi-Ellipsis* A O C, be turn'd about upon the Line A C, it will generate a Solid A G C O, which is call'd a *Spheroid*.

Note, 1. The Content of every *Spheroid* is equal to two thirds of the Content of a Cylinder, whose Base is equal to the greatest Circle of the *Spheroid*; and its Altitude equal to the *Spheroid's* Axis. Therefore the Area of the greatest Circle of a *Spheroid*, multiplied by two thirds of its Axis gives the Content.

2. If the *Spheroid*, be cut by a Plain Z Z, parallel to the Line O G, and through the middle of B C, the Content of the Frustrum Z C Z, to the Content of a Cylinder, whose Base is Z Z, and Altitude V C, is as 5 to 9. Therefore, the Area of the Base Z Z, multiplied by five Ninths of V C, gives the Content of the Frustrum Z C Z.

3. If the *Spheroid* be cut as above, the upper Frustrum ZOAHGZ, is equal to a Cylinder, whose Base is Z Z, and its Altitude V A. Therefore the Area of the Base Z Z, multiplied by the Altitude V A, gives the Content of the Frustrum Z O A G Z. These three Propositions do also hold in a *Sphere*.
Vid. Dr. Wallis's Treatise of Algebra, Page 312.

Problem

Problem VIII. Fig. 7.

To describe a Spheroid, the middle Frustum of which shall represent a Cask, whose Bung Diameter OG , is 32 Inches, the Diameters, at the Heads OH and OK , are each 24 Inches, and the Length EE 40 Inches. And to find the Content of such Cask upon every Inch of the Depth EE .

From the Square of (OG) the Bung-Diameter, Subtract the Square (OH) the Head-Diameter. Then,

As that Difference,

Is to the Square of OG ,

So is the Square of BE ,

To the Square of BA .

The Square-Root of BA , is the Semi-Axis of the Spheroid, with which, as Radius, describe the Circle $FALC$, then find the Points oo , as in the last Problem, and so the Diameters OH . Oh , as many as you please, one in the middle of every Inch of EB , will be sufficient: For the Area's of these Circles in Gallons, being added together, gives the Content of the Part $O'GHo$, and this doubled is the Content of the Cask $OHKO$.

But for our present purpose it will be sufficient to find the Square of a Diameter in the middle of each of the three first Inches, from B towards K , the Squares of the rest, and also the Areas of their

their Circles being easily obtained by their Common Difference. Thus,

From the Square of (OG) the Bung Diameter, subtract the Square of (OH) the Head Diameter, and divide the Square Root of the Difference by (BE) the Semi-Length of the Cask. Then multiply this Quotient by .5, 1.5, and 2.5, and call the Products S.

Now, if from the Square of (OG) the Bung Diameter, you subtract the Squares of S, the Remainders will be the Squares of the respective Diameters sought.

$$\begin{array}{rcl} \text{OG} & \equiv & 32 \\ \text{OR} & \equiv & 24 \\ \text{Square is} & \equiv & 1024 \\ \text{Square is} & \equiv & 576 \\ \hline \text{Difference} & = & 448 \end{array}$$

The Square Root of 448, is 21.16601, this divided by (BE) 20, gives 1.0583, which multiplied

$$\text{by } \left. \begin{array}{l} .5 \\ 1.5 \\ 2.5 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} .52915 \\ 1.58745 \\ 2.64575 \end{array} \right\} \begin{array}{l} = \text{to S.} \\ \text{which} \\ \text{Squared is} \end{array} \left\{ \begin{array}{l} .28 \\ 2.52 \\ 7.00 \end{array} \right.$$

Now the Square of OG, viz. 1024,

$$\begin{array}{l} \text{Less the} \\ \text{Square} \\ \text{of S. viz.} \end{array} \left\{ \begin{array}{l} .28 \\ 2.52 \\ 7.00 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 1023.72 \\ 1021.48 \\ 1017.00 \end{array} \right.$$

which are the Squares of the three first Diameters, one in the middle of every Inch from OG, towards O K, or OH; and these multiplied severally

verally by .0027851, gives the Area's of the respective Circles in Ale-Gallons.

$$\begin{array}{r} \text{Viz.} \left\{ \begin{array}{l} 2.85116 \\ 2.84493 \\ 2.83244 \end{array} \right\} \begin{array}{l} .00624 \\ .01248 \\ .01248 \end{array} \end{array} \quad .00624$$

Now if the second Inch be subtracted from the first, the Difference is .00624, and the third Inch subtracted from the second, the Difference is .01248, then, from .01248, subtract .00624, the difference will be .00624, which is the same throughout, as in the following Table. In which observe, that the first Differences are made by a continual Addition of the second Difference, and the Area's in the second Column are obtain'd by subtracting the first of the first Differences from the first Area, and so on. These Area's added together are 48.724 Gallons equal the Content of the part O G K O, which doubled is 97.448 the Content of the Cask OHK O which was required.

A Table shewing the Content of every Inch of half the middle Frustum of the Spheroid, OOKG, Fig. VII.

	Content of every Inch of the Part OOK O	1. Differ.	2. Differ.
1	2.85116		
2	2.84492	.00624	.00624
3	2.83244	.01248	.00624
4	2.81372	.01872	.00624
5	2.78876	.02496	.00624
6	2.75756	.03120	.00624
7	2.72012	.03744	.00624
8	2.67644	.04368	.00624
9	2.62652	.04992	.00624
10	2.57036	.05616	.00624
11	2.50796	.06240	.00624
12	2.43932	.06864	.00624
13	2.36444	.07488	.00624
14	2.28332	.08112	.00624
15	2.19596	.08736	.00624
16	2.10236	.09360	.00624
17	2.00252	.09984	.00624
18	1.89644	.10608	.00624
19	1.78412	.11232	.00624
20	1.66556	.11856	

Total = 48.724

The Double is 97.448 = Content of the Cask.

O 2

of

Of a Circular Spindle.

IF the Arch of a Circle ZAZ (Fig. 8.) be turn'd about upon its Cord ZBZ , it will describe a Solid $ZOAO$, $ZFCE$, and this Solid, I call a *Circular Spindle*.

Problem IX. Figure 8.

To describe a Circular Spindle, the middle Frustums of which shall represent a Cask whose Bung-Diameter AC , is 32 Inches the Diameters at the Heads OE , and OF are each 24 Inches, and the Length GH 40 Inches. And to find the Content of such Cask in Ale-Gallons.

DIvide the Square of the Semi-Length of the Cask, by half the Difference between the Bung and Head Diameters, and to the Quotient add the said half Difference; the half of this Sum is equal to the Radius AD ; with which (upon D as a Center) describe the Arch LAM , draw the Line LM , and make AD perpendicular thereto. Make AB equal to 16 the Semi-Diameter at the Bung. Draw the Line ZZ parallel to LM and make BC , equal to BA . Through the Points ZCZ , describe the Arch $ZECFZ$, set off the Semi-Length of the Cask from B , to G and H , through which Points, draw Lines parallel

APPENDIX.

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rallel to A D. So shall O E and O F, be the Head Diameters of the Cask, and each 24.

To find a Diameter in the middle of every Inch from A C, towards O E. Put $S = 60$. It is manifest, that the Square of O D, less the Square of S, is equal to the Square of O g. And if from the Square-Root of O g, you take G g, the Remainder G o, will be the Semi-Diameter; which double gives the Diameter O E: : And if we put the first $S = 5$, the second $= 1.5$ the third $= 2.5$, &c. and proceed as here directed, we shall have a Diameter in the middle of every Inch, as in the second Column of the following Table, against which, in the third Column, are the Area's in Ale-Gallons, that is, the Content of so many Circular Plains of one Inch thick, the Sum of these is 48.3938, the Content of the part C E O A; which doubled is 96.7876, the Content of the Cask required.

1	24.0000	1
2	23.9999	4
3	23.9997	9
4	23.9994	16
5	23.9989	25
6	23.9982	36
7	23.9973	49
8	23.9961	64
9	23.9947	81
10	23.9930	100
11	23.9911	121
12	23.9889	144
13	23.9864	169
14	23.9836	196
15	23.9805	225
16	23.9771	256
17	23.9734	289
18	23.9694	324
19	23.9650	361
20	23.9603	400
21	23.9553	441
22	23.9500	484
23	23.9444	529
24	23.9385	576
25	23.9322	625
26	23.9256	676
27	23.9187	729
28	23.9115	784
29	23.9040	841
30	23.8962	900
31	23.8881	961
32	23.8797	1024
33	23.8710	1089
34	23.8620	1156
35	23.8527	1225
36	23.8431	1296
37	23.8332	1369
38	23.8230	1444
39	23.8125	1521
40	23.8017	1600
41	23.7906	1681
42	23.7792	1764
43	23.7675	1849
44	23.7555	1936
45	23.7432	2025
46	23.7306	2116
47	23.7177	2209
48	23.7045	2304
49	23.6910	2401
50	23.6772	2500

O 3

A

A Table shewing the Content of half the middle Frustum of the *Circular Spindle* OECFOA. Fig. VIII.

	Diam. in the middle of every Inch of B. G.	Areas in Ale Gall.
1	31.9952	2.8510
2	31.9567	2.8442
3	31.8797	2.8305
4	31.7642	2.8090
5	31.6098	2.7828
6	31.4166	2.7491
7	31.1843	2.7084
8	30.9126	2.6614
9	30.6012	2.6309
10	30.2497	2.5464
11	29.8577	2.4828
12	29.4248	2.4113
13	28.9505	2.3342
14	28.4340	2.2517
15	27.8749	2.1640
16	27.2723	2.0615
17	26.6255	1.9744
18	25.9336	1.8733
19	25.1957	1.7680
20	24.4016	1.6589

Sum = 48.3938

Doubled is = 96.7876 = Content
of the Cask.

From

From what hath been said, it appears, that the middle Frustum of each of the four Solids last mention'd, may represent a close Cask, (*viz.* a *Butt*, *Pipe*, *Hogshead*, &c.) And if a Cask whose Diameters and Length are as above supposed,

Viz. { Bung } Diameter } 32 }
 { Head } } 24 } Inches.
 { Length } 40 }

Ale-Gall.

I say, if such Cask be the middle Frustum of a *Spheroid* (as OHKO Fig. VII.) the Content (by Prob. VIII.) will be 97.448

If the middle Frustum of a *Circular Spindle*, (as OEFO, Fig. VIII.) the Content (by Probl. IX.) will be 96.787

If the middle Frustum of a *Parabolick Spindle*, (as DCEF, Fig. V.) the Content by Probl. IV. will be 96.498

If the middle Frustum of a *Hyperbolick Spindle* (as OEFO, Fig. VI.) the Content by Problem VI. will be 96.200

And if (as some Writers suppose) the said Cask be the middle Frustum of two *Parabolick Conoids*, as butting upon one common Base, (as ODOOBGO, Fig. III.) the Content by Probl. III. will be 89.122

If the middle Frustum of two *Cones* as butting upon one common Base, the Content will be 87.934

But neither of the two last do often (if at all) occur in the Practice of Gauging.

And though in several Books on this Subject we meet with the Terms *Frustrum*, of a *Spheroid*, *Conoid*, *Parabolick Conoid*, *Parabolick Spindle*, &c. the Writers of such Tracts have not (that I can find) told us what they mean by these Terms, nor will the Rules they have given find the Content of some of these Solids tolerably near the Truth.

For instance,

If the Cask above-mentioned be the middle Frustrum of a *Parabolick Spindle*, the Content (as we have shewn above) is 96.498, Ale-Gallons.

Whereas Mr. *Fordain*, in his *Duodecimal Arithmetick*, makes it but 93.877.

Mr. *Dary*, at the end of his *Miscellanies*, makes the Content but 90.948.

And Mr. *Hunt*, in his *Practical Gauging Epitomized*, makes the Content of this Cask but 87.930, which is above 8 Gallons less than the true Content.

If what has been said in this Appendix shall give any Light to the Practical Gauger, in conceiving the Nature and Properties of the Solids here mentioned, it is as much as was at first designed.

For although the middle Frustrum of each of the four Solids described in Prob. IV, VI, VII, and IX. do very properly represent a Cask, and the Contents there found be sufficiently exact, yet the Contents of three of 'em differ so little one from another, that if every Cask were in the Form of one of

Fig. I.

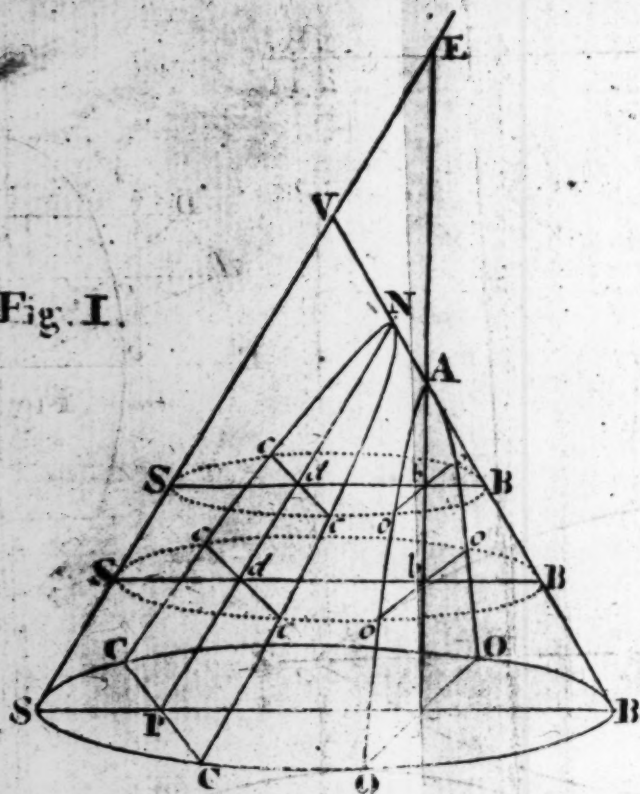


Fig. II.

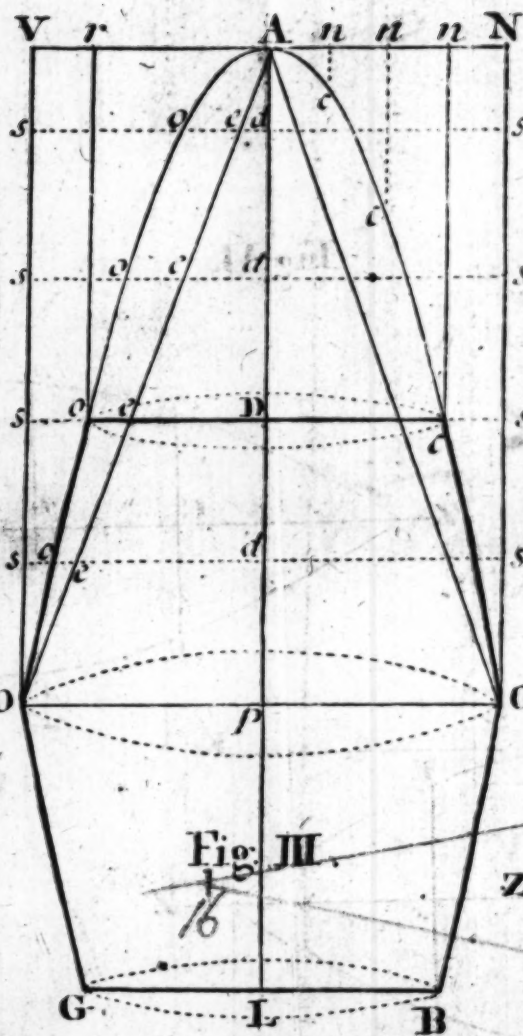
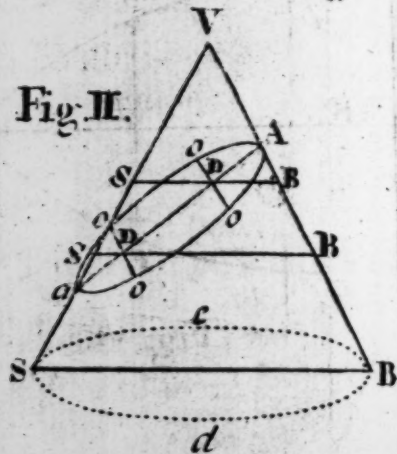
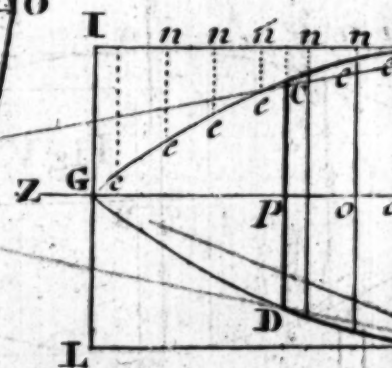
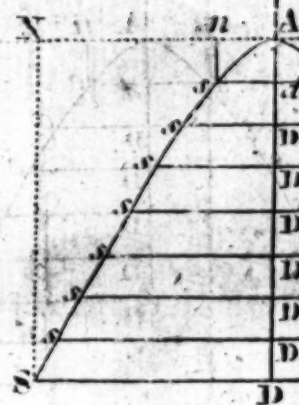


Fig. IV.



Place this at the end of the Appendix.

E

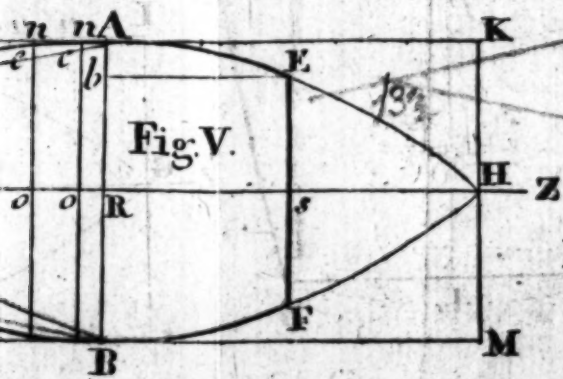
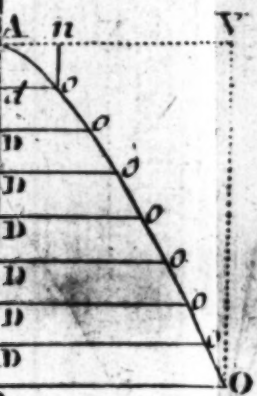


Fig. VI.

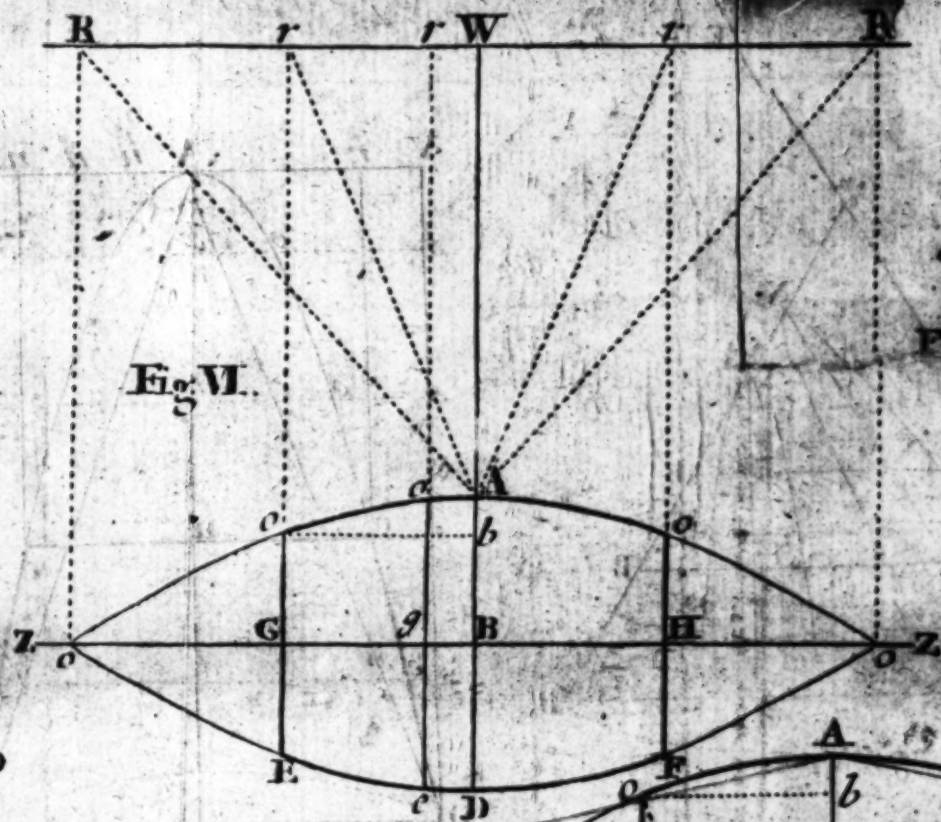
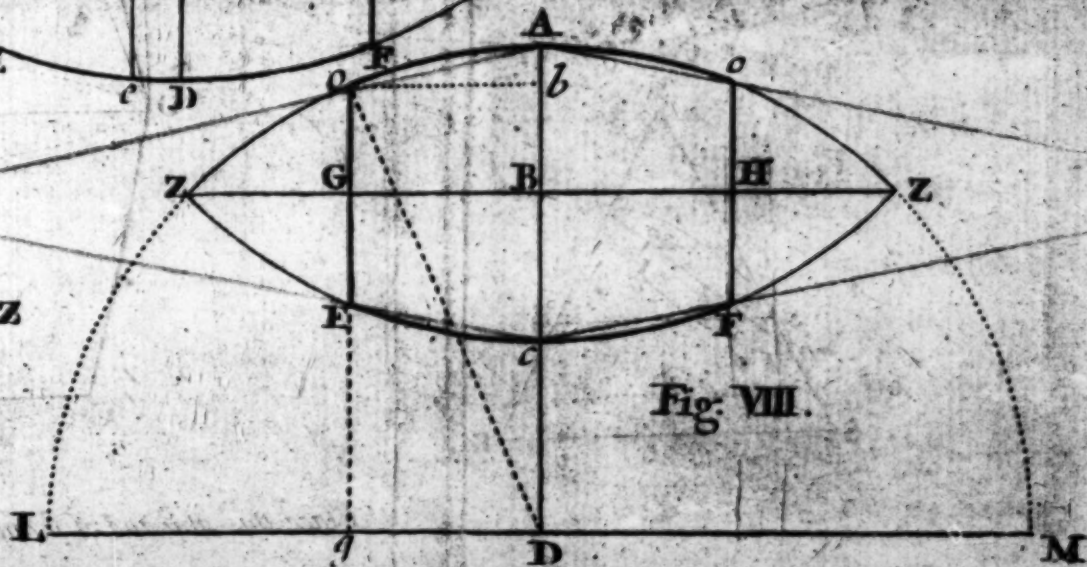


Fig. VIII.



of those three Solids, it would be needless to make any Distinction, because upon sight of a Cask, one could not tell to which of those Solids it agreed.

And there being very few (if any) close Casks of such Diameters and Lengths as we have frequently mention'd, which contain more than the middle Frustum of a *Spheroid* (this having the greatest Curvature between the Head and Bung) But very many Casks (of such Diameters and Length) which will not contain so much as the middle Frustum of a *Parabolick* or *Hyperbolick Spindle*, (their Staves between the Head and Bung being nearer a straight Line than either of these Frustums) I must refer the Practical Gauger to the Rules laid down in the four first Problems of the Ninth Section of the foregoing Treatise, by which the Content of any Cask may be readily found (either by the Pen or Instrument) sufficiently near the Truth.



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F I N I S.

A. D.

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